

EXAMEN ECUACIONES E INECUACIONES

1.- Resuelve los siguientes sistemas de inecuaciones: (4 puntos)

$$\text{a) } \left. \begin{array}{l} \frac{2x-11}{3} - \frac{x+2}{6} \leq 0 \\ 3(1-x)+1 > -5 \end{array} \right\}$$

$$\text{b) } \left. \begin{array}{l} x+3 < 2 \\ 2x+5 \geq 3x \\ x-3 > 5 \end{array} \right\}$$

$$\text{c) } \left. \begin{array}{l} 2x-y \leq 3 \\ x+1 > 0 \end{array} \right\}$$

2.- Resuelve las siguientes inecuaciones y expresa sus soluciones en forma de intervalo:

$$\text{a) } \frac{x-9}{5} - \frac{5x-13}{15} > \frac{4x}{3} + 10 \quad (3 \text{ puntos})$$

$$\text{b) } \frac{x-1}{x+2} \leq 3$$

3.- Resuelve las ecuaciones:

$$\text{a) } \sqrt{7}(x^2 - x)(2x^2 - 8) = 0 \quad (3 \text{ puntos})$$

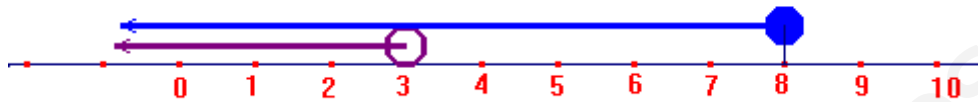
$$\text{b) } \frac{2x}{3} - \frac{x+2}{5} = 1 - (x-3)^2$$

$$\text{c) } \frac{3x}{x^2-5} = \frac{9}{x+1}$$

SOLUCIONES

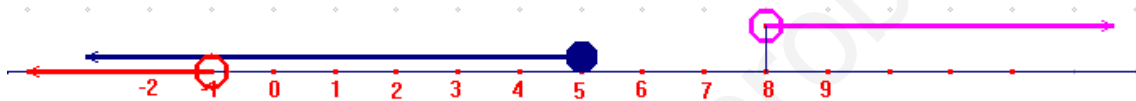
$$1.- a) \left. \begin{array}{l} \frac{2x-11}{3} - \frac{x+2}{6} \leq 0 \\ 3(1-x)+1 > -5 \end{array} \right\} \rightarrow \left. \begin{array}{l} \frac{2(2x-11)}{6} - \frac{x+2}{6} \leq 0 \\ 3-3x+1 > -5 \end{array} \right\} \rightarrow \left. \begin{array}{l} 4x-22-x-2 \leq 0 \\ -3x > -5-3-1 \end{array} \right\}$$

$$\left. \begin{array}{l} 3x \leq 24 \\ 3x < 9 \end{array} \right\} \begin{array}{l} x \leq 8 \\ x < 3 \end{array} \left. \right\} \text{Sol: } (-\infty, 3)$$



b)

$$\left. \begin{array}{l} x+3 < 2 \\ 2x+5 \geq 3x \\ x-3 > 5 \end{array} \right\} \rightarrow \left. \begin{array}{l} x < 2-3 \\ 2x-3x \geq -5 \\ x > 5+3 \end{array} \right\} \rightarrow \left. \begin{array}{l} x < -1 \\ -x \geq -5 \\ x > 8 \end{array} \right\} \rightarrow \left. \begin{array}{l} x < -1 \\ x \leq 5 \\ x > 8 \end{array} \right\} \text{No tiene solución}$$

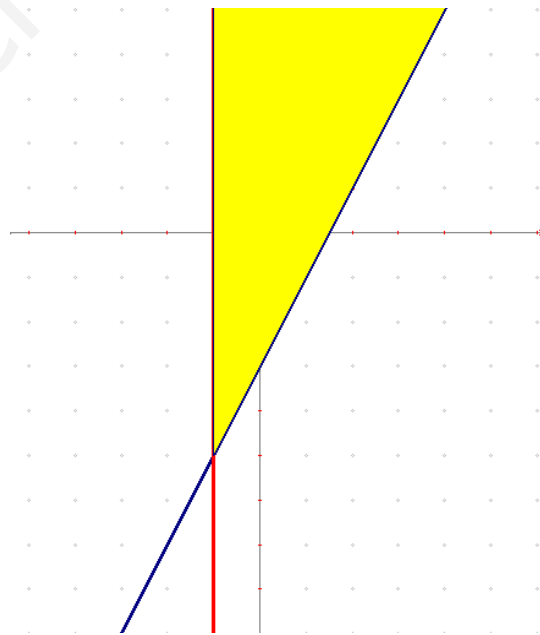


$$c) \left. \begin{array}{l} 2x-y \leq 3 \\ x+1 > 0 \end{array} \right\} \rightarrow \left. \begin{array}{l} 2x-y=3 \\ x+1=0 \end{array} \right\} \rightarrow \left. \begin{array}{l} y=2x-3 \\ x=-1 \end{array} \right\} \text{se representan ambas rectas y se ve}$$

qué semiplano es la solución de cada inecuación, donde coinciden ambos semiplanos es la solución del problema (en amarillo)

$$(0,0) \left. \begin{array}{l} 2 \cdot 0 - 0 \leq 3 \quad \text{SI} \\ 0 + 1 > 0 \quad \text{SI} \end{array} \right\}$$

La semirrecta azul forma parte de la solución



$$2.- a) \frac{x-9}{5} - \frac{5x-13}{15} > \frac{4x}{3} + 10$$

$$\frac{3(x-9)}{15} - \frac{5x-13}{15} > \frac{20x}{15} + \frac{150}{15} \rightarrow 3x-27-5x+13 > 20x+150$$

$$-2x-20x > 150+27-13 \rightarrow -22x > 164 \rightarrow x < \frac{164}{-22} \rightarrow x < -\frac{82}{11}$$

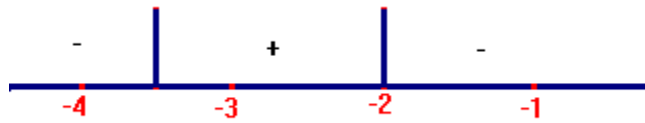
$$\text{Sol: } \left(-\infty, -\frac{82}{11} \right)$$

$$b) \frac{x-1}{x+2} \leq 3 \rightarrow \frac{x-1}{x+2} - 3 \leq 0 \rightarrow \frac{x-1-3x-6}{x+2} \leq 0 \rightarrow \frac{-2x-7}{x+2} \leq 0$$

$$-2x-7=0 \rightarrow x = -\frac{7}{2}$$

$$x+2=0 \rightarrow x = -2$$

$$\text{Sol: } \left(-\infty, -\frac{7}{2}\right] \cup (-2, +\infty)$$



$$3.- a) \sqrt{7}(x^2-x)(2x^2-8)=0 \rightarrow \begin{cases} x^2-x=0 \rightarrow x(x-1)=0 \rightarrow \begin{cases} x=0 \\ x=1 \end{cases} \\ 2x^2-8=0 \rightarrow 2x^2=8 \rightarrow x^2=4 \rightarrow x=\pm 2 \end{cases}$$

$$b) \frac{2x}{3} - \frac{x+2}{5} = 1 - (x-3)^2 \rightarrow \frac{10x}{15} - \frac{3(x+2)}{15} = \frac{15}{15} - \frac{15(x^2-6x+9)}{15}$$

$$10x - 3x - 6 = 15 - 15x^2 + 90x - 135 \rightarrow 15x^2 - 83x + 114 = 0$$

$$x = \frac{83 \pm \sqrt{83^2 - 4 \cdot 15 \cdot 114}}{30} = \frac{83 \pm \sqrt{49}}{30} = \frac{83 \pm 7}{30} = \begin{cases} 3 \\ \frac{76}{30} = \frac{38}{15} \end{cases}$$

$$c) \frac{3x}{x^2-5} = \frac{9}{x+1} \rightarrow 3x(x+1) = 9(x^2-5) \rightarrow 3x^2 + 3x = 9x^2 - 45$$

$$6x^2 - 3x - 45 = 0 \rightarrow 2x^2 - x - 15 = 0 \rightarrow x = \frac{1 \pm \sqrt{1+120}}{4} = \begin{cases} 3 \\ -\frac{5}{2} \end{cases}$$