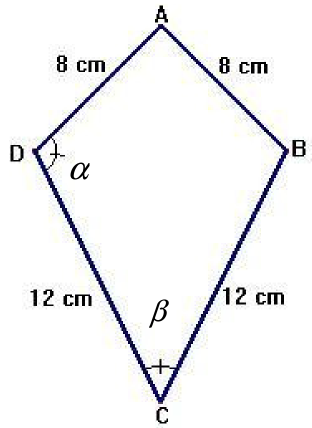
Maths 4th ESO

EXAM 3_1 (Coordinate Geometry-Trigonometry- Surds)

1. In the diagram shown calculate the angles α and β if the diagonal BD has length 10 cm. (1.5 points)



2. Calculate and simplify: (1.5 points)

a) $2\sqrt[3]{a^6b} - 3a^2\sqrt[3]{64b} + 5a\sqrt[3]{a^3b} + a^2\sqrt[3]{125b}$ b) $\sqrt{ab}\cdot\sqrt[3]{a^2b}\cdot\sqrt[4]{ab^3}$

3. Rationalize and simplify: (1.5 points)

a) $\dfrac{\sqrt{a}}{\sqrt{a}+\sqrt{b}}+\dfrac{\sqrt{b}}{\sqrt{a}-\sqrt{b}}$ b) $\dfrac{bc}{\sqrt{a}\cdot\sqrt[4]{b}\cdot\sqrt[8]{c}}$

4. With point A(2,3) and straight line $r: x-3y+2=0$ (1.5 points)

a) Write the equation of a line parallel to **r** and joining the point **A**.
b) Write the equation of a line perpendicular to r and joining the point A.

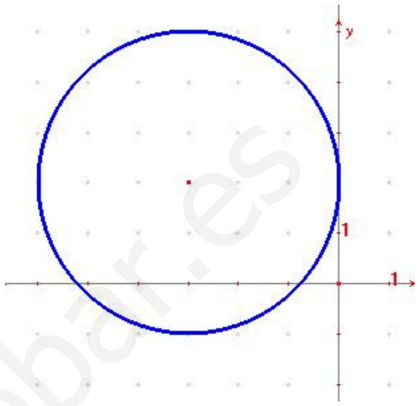
5. Plot these points and label each with the correct letter: (1.5 points)

A = (2, 3) B = (5, 12) C = (10, 5)

a) Draw triangle ABC.
b) Write the coordinates of the midpoint of $\overline{AC}$.
c) Find the length, correct to the nearest hundredth, of the median from B to $\overline{AC}$.
d) Write the equation of the perpendicular bisector of $\overline{AC}$.

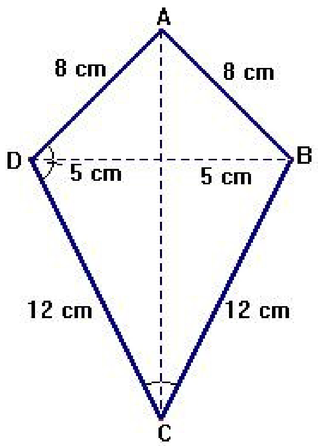
6. Suppose that $cos\,\alpha = -\frac{3}{5}$ and α lies in quadrant II. Draw the angle α and find the other trigonometric ratios for α. (Don't use a calculator) (1.5 points)

7. Write the equation of the circumference in the picture (1 points)



SOLUTION

1. In the diagram shown calculate the angles α and β if the diagonal BD has length 10 cm.



$$\cos\alpha_1 = \frac{5}{8} \Rightarrow \alpha_1 = 51^o19'$$

$$\cos\alpha_2 = \frac{5}{12} \Rightarrow \alpha_2 = 65^o23'$$

$$\alpha = \alpha_1 + \alpha_2 = 51^o19' + 65^o23' = 116^o42'$$

$$\alpha_2 + \frac{\beta}{2} + 90^o = 180^o \rightarrow \frac{\beta}{2} = 90 - 65^o23' = 24^o37'$$

$$\beta = 2\cdot 24^o37' \rightarrow \beta = 49^o14'$$

2. Calculate and simplify:

a)

$$2\sqrt[3]{a^6b} - 3a^2\sqrt[3]{64b} + 5a\sqrt[3]{a^3b} + a^2\sqrt[3]{125b} = 2a^2\sqrt[3]{b} - 3a^2\cdot 2^2\sqrt[3]{b} + 5a^2\sqrt[3]{b} + 5a^2\sqrt[3]{b} = 0$$

b) $\sqrt{ab}\cdot\sqrt[3]{a^2b}\cdot\sqrt[4]{ab^3} = \sqrt[12]{a^6b^6}\cdot\sqrt[12]{a^8b^4}\cdot\sqrt[12]{a^3b^9} = \sqrt[12]{a^{17}b^{19}} = ab\sqrt[12]{a^5b^7}$

3. Rationalize and simplify:

a)

$$\frac{\sqrt{a}}{\sqrt{a}+\sqrt{b}} + \frac{\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)} + \frac{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)} = \frac{a-\sqrt{ab}+\sqrt{ab}+b}{a-b} = \frac{a+b}{a-b}$$

b) $\dfrac{bc}{\sqrt{a}\cdot\sqrt[4]{b}\cdot\sqrt[8]{c}} = \dfrac{bc}{\sqrt[8]{a^4b^2c}} = \dfrac{bc\sqrt[8]{a^4b^6c^7}}{\sqrt[8]{a^4b^2c}\sqrt[8]{a^4b^6c^7}} = \dfrac{bc\sqrt[8]{a^4b^6c^7}}{abc} = \dfrac{\sqrt[8]{a^4b^6c^7}}{a}$

4. With point $A(2,3)$ and straight line $r: x-3y+2=0$

a) Write the equation of a line parallel to **r** and joining the point **A**.

$$r: x-3y+2=0 \rightarrow 3y = x+2 \rightarrow y = \frac{1}{3}x + \frac{2}{3} \Rightarrow m = \frac{1}{3}$$

Parallel line: $y-3 = \frac{1}{3}(x-2) \Rightarrow y = \frac{1}{3}x - \frac{2}{3} + 3 \Rightarrow y = \frac{1}{3}x + \frac{7}{3}$

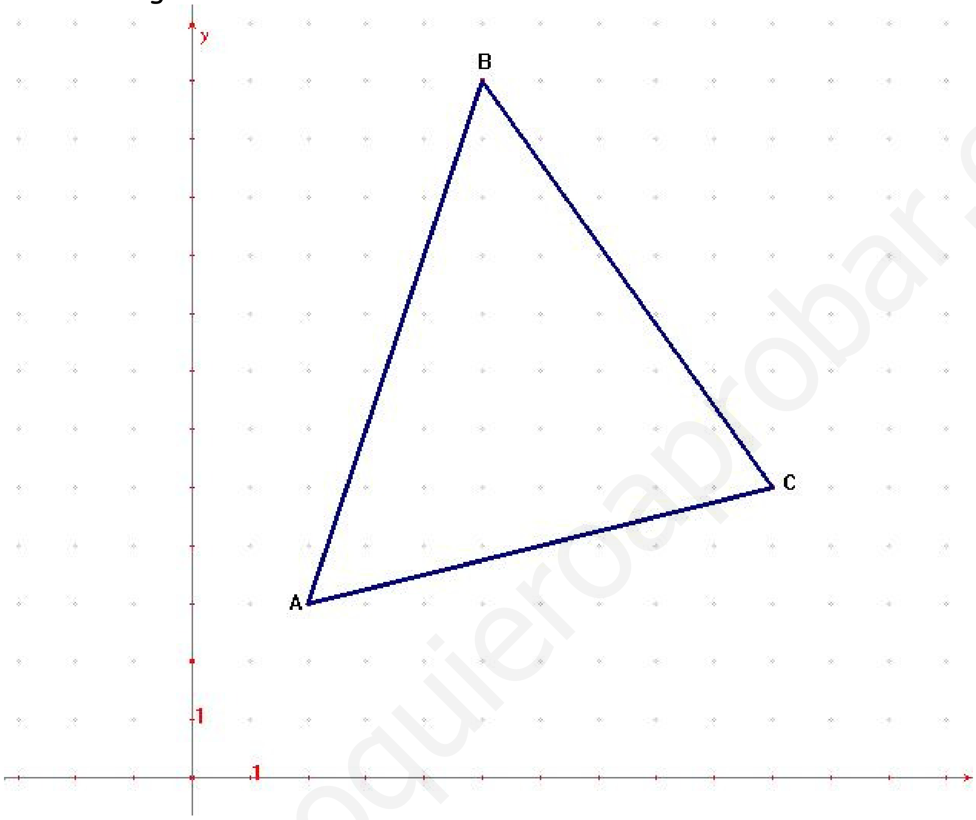
b) Write the equation of a line perpendicular to r and joining the point A.

perpendicular line $\Rightarrow m' = -3 \rightarrow y-3 = -3(x-2) \Rightarrow y = -3x+9$

5. Plot these points and label each with the correct letter:

A = (2, 3) B = (5, 12) C = (10, 5)

a) Draw triangle ABC.



b) Write the coordinates of the midpoint of $\overline{AC}$.

$M\left(\frac{2+10}{2}, \frac{3+5}{2}\right) \rightarrow M(6,4)$

c) Find the length, correct to the nearest hundredth, of the median from B to $\overline{AC}$.

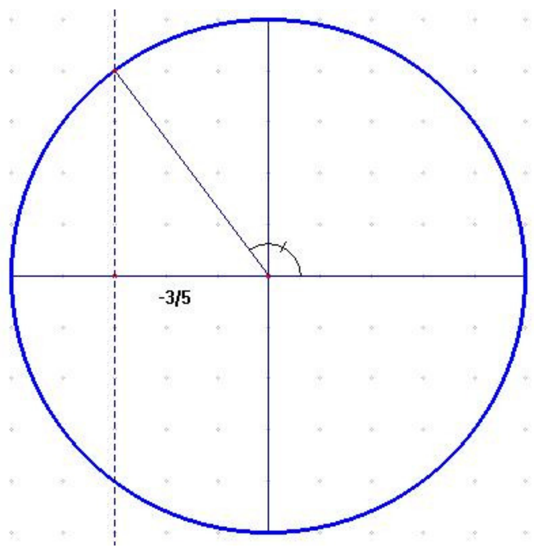
$d(B,M) = \sqrt{(6-5)^2 + (12-4)^2} = \sqrt{1+64} = \sqrt{65} = 8.06$ ul

d) Write the equation of the perpendicular bisector of $\overline{AC}$.

Point M, perpendicular to AC $\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{2-10} = \frac{1}{4} \Rightarrow m' = -4$

$y - 4 = -4(x-6) \Rightarrow y = -4x + 28$

6. Suppose that $cos\,\alpha = -\dfrac{3}{5}$ and α lies in quadrant II. Draw the angle α and find the other trigonometric ratios for α. (Don't use a calculator)



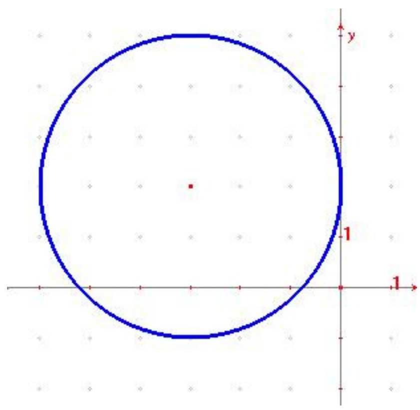
$$cos\,\alpha = -\frac{3}{5};\ sin^2\alpha + cos^2\alpha = 1$$

$$sin^2\alpha = 1 - cos^2\alpha = 1 - \frac{9}{25} = \frac{16}{25}$$

$$sin\,\alpha = \pm\sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$tan\,\alpha = \frac{sin\,\alpha}{cos\,\alpha} = -\frac{4}{3}$$

7. Write the equation of the circumference in the picture



Centre (-3, 2), radius 3

Equation: $(x+3)^2 + (y-2)^2 = 9$