

RADICALES

1. Expresa las siguientes raíces como potencias de exponente fraccionario:

a) $\sqrt[4]{3} = 3^{\frac{1}{4}}$

b) $\sqrt[5]{4^2} = \sqrt[5]{(2^2)^2} = \sqrt[5]{2^4} = 2^{\frac{4}{5}}$

c) $\frac{1}{\sqrt{8}} = \frac{1}{\sqrt{2^3}} = \frac{1}{2^{\frac{3}{2}}} = 2^{-\frac{3}{2}}$

d) $\frac{1}{\sqrt[5]{81}} = \frac{1}{\sqrt[5]{3^4}} = \frac{1}{3^{\frac{4}{5}}} = 3^{-\frac{4}{5}}$

e) $\frac{1}{\sqrt[5]{4^2}} = \frac{1}{\sqrt[5]{(2^2)^2}} = \frac{1}{\sqrt[5]{2^4}} = \frac{1}{2^{\frac{4}{5}}} = 2^{-\frac{4}{5}}$

f) $\frac{1}{\sqrt[3]{8^4}} = \frac{1}{\sqrt[3]{(2^3)^4}} = \frac{1}{\sqrt[3]{2^{12}}} = \frac{1}{2^{\frac{12}{3}}} = 2^{-4}$

g) $\frac{8}{\sqrt[5]{4}} = \frac{2^3}{\sqrt[5]{2^2}} = \frac{2^3}{2^{\frac{2}{5}}} = 2^{\frac{13}{5}}$

h) $\frac{5}{\sqrt[3]{5^2}} = \frac{5^1}{5^{\frac{2}{3}}} = 5^{\frac{1}{3}}$

i) $\frac{1}{\sqrt[3]{243}} = \frac{1}{\sqrt[3]{3^5}} = \frac{1}{3^{\frac{5}{3}}} = 3^{-\frac{5}{3}}$

j) $\sqrt[6]{5^3} = 5^{\frac{3}{6}} = 5^{\frac{1}{2}}$

k) $\sqrt[6]{\sqrt{3}} = \sqrt[12]{3} = 3^{\frac{1}{12}}$

l) $\frac{5}{\sqrt[3]{25}} = \frac{5}{\sqrt[3]{5^2}} = \frac{5^1}{5^{\frac{2}{3}}} = 5^{\frac{1}{3}}$

m) $\frac{1}{\sqrt[3]{9^5}} = \frac{1}{\sqrt[3]{(3^2)^5}} = \frac{1}{\sqrt[3]{3^{10}}} = \frac{1}{3^{\frac{10}{3}}} = 3^{-\frac{10}{3}}$

n) $\frac{2}{\sqrt[3]{16^4}} = \frac{2}{\sqrt[3]{(2^4)^4}} = \frac{2}{\sqrt[3]{2^{16}}} = \frac{2^1}{2^{\frac{16}{3}}} = 2^{-\frac{13}{3}}$

$$o) \frac{3}{\sqrt[5]{9^2}} = \frac{3}{\sqrt[5]{(3^2)^2}} = \frac{3}{\sqrt[5]{3^4}} = \frac{3^1}{3^{\frac{4}{5}}} = 3^{\frac{1}{5}}$$

$$p) \frac{\sqrt[3]{3}}{\sqrt[5]{9^2}} = \frac{3^{\frac{1}{3}}}{\sqrt[5]{3^4}} = \frac{3^{\frac{1}{3}}}{3^{\frac{4}{5}}} = 3^{\frac{1}{3} - \frac{4}{5}} = 3^{-\frac{7}{15}}$$

$$q) \sqrt[3]{\sqrt[6]{64}} = \sqrt[6]{2^6} = 2$$

$$r) \frac{2}{\sqrt{2 \cdot \sqrt[3]{32}}} = \frac{2}{\sqrt{\sqrt[3]{2^3 \cdot 2^5}}} = \frac{2}{\sqrt[6]{2^8}} = \frac{2^1}{2^{\frac{8}{6}}} = \frac{2^1}{2^{\frac{4}{3}}} = 2^{-\frac{1}{3}}$$

$$s) \sqrt[3]{\sqrt{81}} = \sqrt[6]{3^4} = 3^{\frac{4}{6}} = 3^{\frac{2}{3}}$$

$$t) \sqrt[4]{\sqrt{25}} = \sqrt[8]{5^2} = 5^{\frac{2}{8}} = 5^{\frac{1}{4}}$$

$$u) \frac{\sqrt{\sqrt{3}}}{\sqrt{27}} = \frac{\sqrt[4]{3}}{\sqrt{3^3}} = \frac{3^{\frac{1}{4}}}{3^{\frac{3}{2}}} = 3^{\frac{1}{4} - \frac{3}{2}} = 3^{-\frac{5}{4}}$$

$$v) \frac{1}{\sqrt[4]{\sqrt[3]{1024}}} = \frac{1}{\sqrt[12]{2^{10}}} = \frac{1}{2^{\frac{10}{12}}} = 2^{-\frac{5}{6}}$$

$$w) \frac{\sqrt[3]{5 \cdot \sqrt[5]{15}}}{\sqrt{45}} = \frac{\sqrt[3]{5 \cdot 5^{\frac{1}{5}} \cdot 3 \cdot 5}}{\sqrt{3^2 \cdot 5}} = \frac{\sqrt[15]{3 \cdot 5^6}}{3 \cdot 5^{\frac{1}{2}}} = \frac{3^{\frac{1}{15}} \cdot 5^{\frac{6}{15}}}{3^1 \cdot 5^{\frac{1}{2}}} = \frac{3^{\frac{1}{15}} \cdot 5^{\frac{2}{5}}}{3^1 \cdot 5^{\frac{1}{2}}} = 3^{-\frac{14}{15}} \cdot 5^{-\frac{1}{10}}$$

$$x) \frac{36}{\sqrt{6 \cdot \sqrt[3]{27}}} = \frac{2^2 \cdot 3^2}{\sqrt{2 \cdot 3 \cdot \sqrt[3]{3^3}}} = \frac{2^2 \cdot 3^2}{\sqrt{\sqrt[3]{2^3 \cdot 3^3 \cdot 3^3}}} = \frac{2^2 \cdot 3^2}{\sqrt[6]{2^3 \cdot 3^6}} = \frac{2^2 \cdot 3^2}{2^{\frac{1}{2}} \cdot 3^1} = 2^{\frac{3}{2}} \cdot 3$$

$$y) \sqrt{12 \cdot \sqrt[3]{18}} = \sqrt{2^2 \cdot 3 \cdot \sqrt[3]{2 \cdot 3^2}} = \sqrt{\sqrt[3]{2^6 \cdot 3^3 \cdot 2 \cdot 3^2}} = \sqrt[6]{2^7 \cdot 3^5} = 2^{\frac{7}{6}} \cdot 3^{\frac{5}{6}}$$

$$z) \frac{1}{\sqrt[3]{81^{-5}}} = \frac{1}{\sqrt[3]{(3^4)^{-5}}} = \frac{1}{\sqrt[3]{3^{-20}}} = \frac{1}{3^{-\frac{20}{3}}} = 3^{\frac{20}{3}}$$

2. Extrae todos los factores posibles de los siguientes radicales:

$$a) \sqrt{9a^4bc^5} = \sqrt{3^2 a^4 bc^5} = 3a^2 c^2 \sqrt{bc}$$

$$b) \sqrt{18a^{12}b^9c^7} = \sqrt{2 \cdot 3^2 \cdot a^{12}b^9c^7} = 3a^6b^4c^3 \sqrt{2bc}$$

$$c) \sqrt[3]{864a^4b^9c^{24}} = \sqrt[3]{2^5 \cdot 3^3 \cdot a^4b^9c^{24}} = 2 \cdot 3 \cdot a \cdot b^3 \cdot c^8 \sqrt[3]{2^2 a} = 6ab^3c^8 \sqrt[3]{4a}$$

$$d) \sqrt[7]{3072a^{12}b^{14}c^{26}} = \sqrt[7]{2^{10} \cdot 3 \cdot a^{12}b^{14}c^{26}} = 2 \cdot a \cdot b^2 \cdot c^3 \sqrt[7]{2^3 \cdot 3 \cdot a^5 c^5} = 2ab^2c^3 \sqrt[7]{24a^5c^5}$$

$$e) \sqrt{\frac{8x^3y}{125z^9}} = \sqrt{\frac{2^3x^3y}{5^3z^9}} = \frac{2x}{5z^4} \sqrt{\frac{2xy}{5z}}$$

$$f) \sqrt[3]{\frac{108a^9bc^{16}}{875d^4e}} = \sqrt[3]{\frac{2^2 \cdot 3^3 \cdot a^9bc^{16}}{5^3 \cdot 7 \cdot d^4e}} = \frac{3a^3c^5}{5d} \sqrt[3]{\frac{2^2bc}{7de}} = \frac{3a^3c^5}{5d} \sqrt[3]{\frac{4bc}{7de}}$$

$$g) \sqrt{\frac{16a^4}{27bc^6}} = \sqrt{\frac{2^4a^4}{3^3bc^6}} = \frac{2^2a^2}{3c^3} \sqrt{\frac{1}{3b}} = \frac{4a^2}{3c^3} \sqrt{\frac{1}{3b}}$$

$$h) \sqrt[3]{\frac{8a^9c^{10}}{125d^3e^9}} = \sqrt[3]{\frac{2^3a^9c^{10}}{5^3d^3e^9}} = \frac{2a^3c^3}{5de^3} \sqrt[3]{c}$$

3. Introduce en el radical los factores que multiplican a las raíces siguientes:

$$a) 3a^2b\sqrt{ab} = \sqrt{3^2a^4b^2ab} = \sqrt{9a^5b^3}$$

$$b) pq^3 \cdot \sqrt{3pq} = \sqrt{p^2q^6 \cdot 3pq} = \sqrt{3p^3q^7}$$

$$c) 2a^3b \cdot \sqrt[3]{3a^2b} = \sqrt[3]{2^3a^9b^3 \cdot 3a^2b} = \sqrt[3]{24a^{11}b^4}$$

$$d) \frac{2}{5}xy^2 \sqrt{\frac{2xy}{5}} = \sqrt{\frac{2^2 \cdot x^2 \cdot y^4 \cdot 2xy}{5^2 \cdot 5}} = \sqrt{\frac{8x^3y^5}{125}}$$

$$e) \frac{2x^2a}{y^3} \sqrt[5]{\frac{2ax}{5y^2}} = \sqrt[5]{\frac{2^5 \cdot x^{10} \cdot a^5 \cdot 2ax}{y^{15} \cdot 5y^2}} = \sqrt[5]{\frac{64x^{11}a^6}{5y^{17}}}$$

$$f) 2ab \cdot \sqrt[4]{3a^2b} = \sqrt[4]{2^4 \cdot a^4 \cdot b^4 \cdot 3a^2b} = \sqrt[4]{48a^6b^5}$$

$$g) \frac{10x^3c}{9e} \sqrt{\frac{5c}{3}} = \sqrt{\frac{10^2 \cdot x^6 \cdot c^2 \cdot 5c}{9^2 \cdot e^2 \cdot 3}} = \sqrt{\frac{500x^6c^3}{243e^2}}$$

4. Escribe tres radicales equivalentes a los siguientes:

$$a) \sqrt[3]{2} = \sqrt[6]{2^2} = \sqrt[9]{2^3} = \sqrt[12]{2^4}$$

$$b) \sqrt{5} = \sqrt[4]{5^2} = \sqrt[6]{5^3} = \sqrt[8]{5^4}$$

$$c) \sqrt[5]{-5} = -\sqrt[10]{(-5)^2} = \sqrt[15]{(-5)^3} = -\sqrt[20]{(-5)^4}$$

$$d) \sqrt[4]{3^6} = \sqrt[8]{3^{12}} = \sqrt[12]{3^{18}} = \sqrt[16]{3^{24}}$$

$$e) \sqrt[5]{(-2)^3} = -\sqrt[10]{(-2)^6} = \sqrt[15]{(-2)^9} = -\sqrt[20]{(-2)^{12}}$$

$$f) \sqrt{8} = \sqrt[4]{8^2} = \sqrt[6]{8^3} = \sqrt[8]{8^4}$$

5. Simplifica los siguientes radicales:

a) $\sqrt[36]{2^{45}} = \sqrt[4]{2^5} = \sqrt[4]{32} \quad m.c.d.(36,45) = 9$

b) $\sqrt[12]{625a^8b^{20}} = \sqrt[12]{5^4a^8b^{20}} = \sqrt[3]{5a^2b^5} \quad m.c.d.(12,4,8,20) = 4$

c) $\sqrt[6]{512} = \sqrt[6]{2^9} = \sqrt{2^3} = \sqrt{8} \quad m.c.d.(6,9) = 3$

d) $\sqrt[60]{3^{84}b^{12}} = \sqrt[5]{3^7b} = \sqrt[5]{2187b} \quad m.c.d.(60,84,12) = 12$

e) $\sqrt[12]{1728} = \sqrt[12]{2^6 \cdot 3^3} = \sqrt[4]{2^2 \cdot 3} = \sqrt[4]{12} \quad m.c.d.(12,6,3) = 3$

f) $\sqrt[4]{324a^6b^{14}} = \sqrt[4]{2^2 \cdot 3^4 \cdot a^6b^{14}} = \sqrt{2 \cdot 3^2 \cdot a^3b^7} = \sqrt{18a^3b^7} \quad m.c.d.(4,2,6,14) = 2$

6. Calcula y simplifica:

a) $\sqrt[3]{2} \cdot \sqrt[3]{16} \cdot \sqrt[3]{128} = \sqrt[3]{2} \cdot \sqrt[3]{2^4} \cdot \sqrt[3]{2^7} = \sqrt[3]{2 \cdot 2^4 \cdot 2^7} = \sqrt[3]{2^{12}} = 2^4 = 16$

b) $\sqrt[3]{18} \cdot \sqrt[5]{4} \cdot \sqrt[6]{36} = \sqrt[3]{2 \cdot 3^2} \cdot \sqrt[5]{2^2} \cdot \sqrt[6]{2^2 \cdot 3^2} = \sqrt[30]{2^{10} \cdot 3^{20}} \cdot \sqrt[30]{2^{12} \cdot 3^{10}} = \sqrt[30]{2^{32} \cdot 3^{30}} = \sqrt[15]{2^{16} \cdot 3^{15}} = 2 \cdot 3^{15/15} = 6^{15/15} = 6$

c) $\sqrt[5]{1000} \cdot \sqrt[4]{100} \cdot \sqrt{1000} = \sqrt[5]{10^3} \cdot \sqrt[4]{10^2} \cdot \sqrt{10^3} = \sqrt[20]{10^{12}} \cdot \sqrt[20]{10^{10}} \cdot \sqrt[20]{10^{30}} = \sqrt[20]{10^{52}} = \sqrt[5]{10^{13}} = 10^2 \cdot \sqrt[5]{10^3} = 100 \cdot \sqrt[5]{1000}$

d) $\sqrt[3]{2} \cdot \sqrt{8} \cdot \sqrt[4]{32} = \sqrt[3]{2} \cdot \sqrt{2^3} \cdot \sqrt[4]{2^5} = 2^{1/3} \cdot 2^{3/2} \cdot 2^{5/4} = 2^{1/3 + 3/2 + 5/4} = 2^{37/12} = \sqrt[12]{2^{37}} = 2^3 \cdot \sqrt[12]{2} = 8 \sqrt[12]{2}$

e) $\frac{\sqrt[3]{24}}{\sqrt[3]{3}} = \sqrt[3]{24 : 3} = \sqrt[3]{8} = 2$

f) $\frac{\sqrt{3}}{\sqrt[3]{9}} = \frac{\sqrt{3}}{\sqrt[3]{3^2}} = \frac{\sqrt[6]{3^3}}{\sqrt[6]{3^4}} = \sqrt[6]{3^{-1}} = \sqrt[6]{\frac{1}{3}} = \frac{1}{\sqrt[6]{3}} \stackrel{\text{racionalizar}}{=} \frac{1 \cdot \sqrt[6]{3^5}}{\sqrt[6]{3} \cdot \sqrt[6]{3^5}} = \frac{\sqrt[6]{243}}{3}$

g) $\frac{\sqrt[3]{9}}{\sqrt{3}} = \frac{\sqrt[3]{3^2}}{\sqrt{3}} = \frac{\sqrt[6]{3^4}}{\sqrt[6]{3^3}} = \sqrt[6]{3}$

h) $\frac{\sqrt[3]{25}}{\sqrt[4]{25}} = \frac{\sqrt[3]{5^2}}{\sqrt[4]{5^2}} = \frac{\sqrt[12]{5^8}}{\sqrt[12]{5^6}} = \sqrt[12]{5^2} = \sqrt[6]{5}$

i) $\frac{\sqrt[6]{108}}{\sqrt[9]{288}} = \frac{\sqrt[6]{2^2 \cdot 3^3}}{\sqrt[9]{2^5 \cdot 3^2}} = \frac{\sqrt[18]{2^6 \cdot 3^9}}{\sqrt[18]{2^{10} \cdot 3^4}} = \sqrt[18]{\frac{3^5}{2^4}} = \sqrt[18]{\frac{243}{16}}$

j) $\frac{\sqrt[3]{1200} \cdot \sqrt[5]{125}}{\sqrt[15]{1440}} = \frac{\sqrt[3]{2^4 \cdot 3 \cdot 5^2} \cdot \sqrt[5]{5^3}}{\sqrt[15]{2^5 \cdot 3^2 \cdot 5}} = \frac{\sqrt[15]{2^{20} \cdot 3^5 \cdot 5^{10}} \cdot \sqrt[15]{5^9}}{\sqrt[15]{2^5 \cdot 3^2 \cdot 5}} = \frac{\sqrt[15]{2^{20} \cdot 3^5 \cdot 5^{19}}}{\sqrt[15]{2^5 \cdot 3^2 \cdot 5}} = \sqrt[15]{2^{15} \cdot 3^3 \cdot 5^{18}} \stackrel{\text{simplificar}}{=} 2 \cdot 3 \cdot 5^6 = 2 \cdot 3 \cdot 5^6 = 10^5 \sqrt[15]{15}$

$$k) \frac{4\sqrt{x} \cdot 2^3 \sqrt[3]{x^2}}{\sqrt[6]{x^5}} = \frac{8 \cdot \sqrt[6]{x^3} \cdot \sqrt[6]{x^4}}{\sqrt[6]{x^5}} = 8 \sqrt[6]{x^2} \stackrel{\text{simplificar}}{=} 8 \sqrt[3]{x}$$

$$l) \frac{\sqrt{12x^3} \cdot \sqrt[4]{9y^2}}{\sqrt[6]{4xy}} = \frac{\sqrt{2^2 \cdot 3 \cdot x^3} \cdot \sqrt[4]{3^2 y^2}}{\sqrt[6]{2^2 xy}} = \sqrt[12]{\frac{2^{12} \cdot 3^6 \cdot x^{18} \cdot 3^6 \cdot y^6}{2^4 \cdot x^2 \cdot y^2}} = \sqrt[12]{\frac{2^{12} \cdot 3^{12} \cdot x^{18} \cdot y^6}{2^4 \cdot x^2 \cdot y^2}} = \sqrt[12]{2^8 \cdot 3^{12} \cdot x^{16} \cdot y^4} =$$

$$= \sqrt[3]{2^2 \cdot 3^3 \cdot x^4 \cdot y} = 3x \sqrt[3]{4xy}$$

$$m) \frac{\sqrt[3]{\sqrt{xy}} \cdot \sqrt{8x^3 y}}{\sqrt[3]{2xy}} = \frac{\sqrt[6]{xy} \cdot \sqrt{2^3 x^3 y}}{\sqrt[3]{2xy}} = \sqrt[6]{\frac{x \cdot y \cdot 2^9 \cdot x^9 \cdot y^3}{2^2 \cdot x^2 \cdot y^2}} = \sqrt[6]{2^7 \cdot x^8 \cdot y^2} = 2x \sqrt[6]{2x^2 y^2}$$

$$n) \frac{\sqrt{\sqrt{2^{-3} x a^5}} \cdot \sqrt[3]{128 a^2}}{\sqrt[4]{\sqrt[3]{\frac{1}{8} a^3 x} \cdot \sqrt{16 x^{-3}}}} = \frac{\sqrt{2^{-3} x a^5} \cdot \sqrt[3]{2^7 a^2}}{\sqrt[4]{2^{-3} a^3 x \cdot \sqrt{2^4 x^{-3}}}} = \sqrt[12]{\frac{2^{-9} x^3 a^{15} 2^{28} a^8}{2^{-3} a^3 x 2^{24} x^{-18}}} = \sqrt[12]{\frac{2^{19} x^3 a^{23}}{2^{21} a^3 x^{-17}}} = \sqrt[12]{\frac{2^{19} x^3 a^{23} x^{17}}{2^{21} a^3}} =$$

$$= \sqrt[12]{\frac{2^{19} x^{20} a^{23}}{2^{21} a^3}} = \sqrt[12]{\frac{x^{20} a^{20}}{2^2}} = \sqrt[6]{\frac{x^{10} a^{10}}{2}} = xa = \sqrt[6]{\frac{x^4 a^4}{2}}$$

$$o) \sqrt[4]{125^3} \cdot \sqrt[3]{-5^8} = -4\sqrt[4]{5^9} \cdot \sqrt[3]{5^8} = -12\sqrt[4]{5^{27}} \cdot \sqrt[3]{5^{32}} = -12\sqrt[4]{5^{59}} = -5^4 \sqrt[12]{5^{11}} = -625 \sqrt[12]{5^{11}}$$

$$p) \sqrt[3]{125^5} \cdot \sqrt[7]{-\frac{1}{5^6}} = -\sqrt[3]{5^{15}} \cdot \sqrt[7]{5^{-6}} = -2\sqrt[3]{5^{105}} \cdot 5^{-18} = -2\sqrt[3]{5^{87}} = -5^4 \sqrt[21]{5^3} = -625 \sqrt[21]{125}$$

$$q) \sqrt[3]{a^2} \cdot \sqrt[4]{a^3 \cdot b^{-3}} \cdot \sqrt[3]{-a} = -\sqrt[12]{a^8} \cdot \sqrt[12]{a^9 \cdot b^{-9}} \cdot \sqrt[12]{a^4} = -\sqrt[12]{a^{21} \cdot b^{-9}} = -\sqrt[12]{\frac{a^{21}}{b^9}} = -\sqrt[4]{\frac{a^7}{b^3}} = -a \sqrt[4]{\frac{a^3}{b^3}}$$

$$r) 3\sqrt{2} \cdot \sqrt[3]{3} = 3\sqrt[6]{2^3 \cdot 3^2} = 3\sqrt[6]{72}$$

$$s) \frac{\sqrt{27} \cdot \sqrt[6]{6}}{\sqrt{3}} = \frac{\sqrt{3^3} \cdot \sqrt[6]{2 \cdot 3}}{\sqrt{3}} = \sqrt[12]{\frac{3^9 \cdot 2^2 \cdot 3^2}{3^6}} = \sqrt[12]{\frac{3^{11} \cdot 2^2}{3^6}} = \sqrt[12]{3^5 \cdot 2^2} = \sqrt[12]{243 \cdot 4} = \sqrt[12]{972}$$

$$t) \frac{\sqrt{a} \cdot \sqrt[3]{a} \cdot \sqrt[4]{a^3}}{\sqrt[12]{a^5}} = \frac{\sqrt[3]{\sqrt{a^3} \cdot a} \cdot \sqrt[4]{a^3}}{\sqrt[12]{a^5}} = \frac{\sqrt[6]{a^4} \cdot \sqrt[4]{a^3}}{\sqrt[12]{a^5}} = \sqrt[12]{\frac{a^8 \cdot a^9}{a^5}} = \sqrt[12]{a^{12}} = a$$

$$u) \frac{\sqrt{6048x^7 y^3}}{\sqrt[3]{7938xy^4}} = \frac{\sqrt{2^5 3^3 7 x^7 y^3}}{\sqrt[3]{2 \cdot 3^4 7^2 xy^4}} = \sqrt[6]{\frac{2^{15} 3^9 7^3 x^{21} y^9}{2^2 3^8 7^4 x^2 y^8}} = \sqrt[6]{\frac{2^{13} \cdot 3 \cdot x^{19} y}{7}} = 2^2 x^3 \sqrt[12]{\frac{2 \cdot 3 \cdot x \cdot y}{7}} = 4x^3 \sqrt[12]{\frac{6xy}{7}}$$

$$v) \frac{(\sqrt[3]{a^2})^4 \cdot (a^2 \cdot \sqrt{a})^3}{\sqrt[6]{a^5}} = \frac{\sqrt[3]{a^8} \cdot a^6 \cdot \sqrt{a^3}}{\sqrt[6]{a^5}} = \frac{a^6 \cdot \sqrt[6]{a^{16}} \cdot a^9}{\sqrt[6]{a^5}} = a^6 \sqrt[6]{a^{20}} = a^6 \sqrt[3]{a^{10}} = a^6 \cdot a^3 \sqrt[3]{a} = a^9 \sqrt[3]{a}$$

7. Calcula y simplifica:

$$a) 3\sqrt{2} - 5\sqrt{8} + 7\sqrt{32} = 3\sqrt{2} - 5\sqrt{2^3} + 7\sqrt{2^5} = 3\sqrt{2} - 5 \cdot 2\sqrt{2} + 7 \cdot 2^2\sqrt{2} = 3\sqrt{2} - 10\sqrt{2} + 28\sqrt{2} = 21\sqrt{2}$$

$$b) -2\sqrt{48} - \frac{1}{3}\sqrt{243} - \frac{1}{5}\sqrt{75} = -2\sqrt{2^4 \cdot 3} - \frac{1}{3}\sqrt{3^5} - \frac{1}{5}\sqrt{5^2 \cdot 3} = -2 \cdot 2^2\sqrt{3} - \frac{1}{3} \cdot 3^2\sqrt{3} - \frac{1}{5} \cdot 5\sqrt{3} =$$

$$= -8\sqrt{3} - 3\sqrt{3} - 1\sqrt{3} = -12\sqrt{3}$$

$$c) 5\sqrt{48} - \frac{1}{8}\sqrt{12} + \frac{3}{5}\sqrt{75} = 5\sqrt{2^4 \cdot 3} - \frac{1}{8}\sqrt{2^2 \cdot 3} + \frac{3}{5}\sqrt{5^2 \cdot 3} = 5 \cdot 2^2\sqrt{3} - \frac{1}{8} \cdot 2\sqrt{3} + \frac{3}{5} \cdot 5\sqrt{3} =$$

$$= 20\sqrt{3} - \frac{1}{4}\sqrt{3} + 3\sqrt{3} = \frac{91}{4}\sqrt{3}$$

$$d) \sqrt{2} + \sqrt{8} - 5\sqrt{18} + \sqrt{32} = \sqrt{2} + \sqrt{2^3} - 5\sqrt{2 \cdot 3^2} + \sqrt{2^5} = \sqrt{2} + 2\sqrt{2} - 5 \cdot 3\sqrt{2} + 2^2\sqrt{2} =$$

$$= \sqrt{2} + 2\sqrt{2} - 15\sqrt{2} + 4\sqrt{2} = -8\sqrt{2}$$

$$e) \sqrt{28} + 3\sqrt{63} - 5\sqrt{7} + \sqrt{175} = \sqrt{2^2 \cdot 7} + 3\sqrt{3^2 \cdot 7} - 5\sqrt{7} + \sqrt{5^2 \cdot 7} = 2\sqrt{7} + 3 \cdot 3\sqrt{7} - 5\sqrt{7} + 5\sqrt{7} =$$

$$= 2\sqrt{7} + 9\sqrt{7} - 5\sqrt{7} + 5\sqrt{7} = 11\sqrt{7}$$

$$f) \sqrt{24} + 7\sqrt{6} - 2\sqrt{486} = \sqrt{2^3 \cdot 3} + 7\sqrt{2 \cdot 3} - 2\sqrt{2 \cdot 3^5} = 2\sqrt{2 \cdot 3} + 7\sqrt{2 \cdot 3} - 2 \cdot 3^2\sqrt{2 \cdot 3} =$$

$$= 2\sqrt{6} + 7\sqrt{6} - 18\sqrt{6} = -9\sqrt{6}$$

$$g) \sqrt[3]{108} - 2\sqrt[3]{32} - \frac{1}{3}\sqrt[3]{500} = \sqrt[3]{2^2 \cdot 3^3} - 2\sqrt[3]{2^5} - \frac{1}{3}\sqrt[3]{2^2 \cdot 5^3} = 3\sqrt[3]{2^2} - 2 \cdot 2\sqrt[3]{2^2} - \frac{1}{3} \cdot 5\sqrt[3]{2^2} =$$

$$= 3\sqrt[3]{4} - 4\sqrt[3]{4} - \frac{5}{3}\sqrt[3]{4} = -\frac{8}{3}\sqrt[3]{4}$$

$$h) \sqrt{\frac{2}{3}} + \frac{3}{2}\sqrt{\frac{8}{27}} - \frac{1}{2}\sqrt{\frac{32}{75}} = \sqrt{\frac{2}{3}} + \frac{3}{2}\sqrt{\frac{2^3}{3^3}} - \frac{1}{2}\sqrt{\frac{2^5}{3 \cdot 5^2}} = \sqrt{\frac{2}{3}} + \frac{3}{2} \cdot \frac{2}{3}\sqrt{\frac{2}{3}} - \frac{1}{2} \cdot \frac{2^2}{5}\sqrt{\frac{2}{3}} = \sqrt{\frac{2}{3}} + \sqrt{\frac{2}{3}} - \frac{2}{5}\sqrt{\frac{2}{3}} = \frac{8}{5}\sqrt{\frac{2}{3}}$$

$$i) \frac{3}{5}\sqrt[3]{\frac{8}{81}} - \frac{1}{2}\sqrt[3]{\frac{27}{24}} + 4\sqrt[3]{\frac{1}{64}} = \frac{3}{5}\sqrt[3]{\frac{2^3}{3^4}} - \frac{1}{2}\sqrt[3]{\frac{3^3}{2^3 \cdot 3}} + 4\sqrt[3]{\frac{1}{2^6}} = \frac{3}{5} \cdot \frac{2}{3}\sqrt[3]{\frac{1}{3}} - \frac{1}{2} \cdot \frac{3}{2}\sqrt[3]{\frac{1}{3}} + 4 \cdot \frac{1}{2^2} =$$

$$= \frac{2}{5}\sqrt[3]{\frac{1}{3}} - \frac{3}{4}\sqrt[3]{\frac{1}{3}} + 1 = -\frac{7}{20}\sqrt[3]{\frac{1}{3}} + 1$$

$$j) \sqrt[3]{16} - 2\sqrt[3]{250} - \frac{1}{3}\sqrt[3]{54} = \sqrt[3]{2^4} - 2\sqrt[3]{2 \cdot 5^3} - \frac{1}{3}\sqrt[3]{2 \cdot 3^3} = 2\sqrt[3]{2} - 2 \cdot 5\sqrt[3]{2} - \frac{1}{3} \cdot 3 \cdot \sqrt[3]{2} =$$

$$= 2\sqrt[3]{2} - 10\sqrt[3]{2} - 1\sqrt[3]{2} = -9\sqrt[3]{2}$$

$$k) \sqrt{48} - \frac{\sqrt{128}}{3} + \frac{3\sqrt{3}}{4} - \sqrt{\frac{8}{9}} = \sqrt{2^4 \cdot 3} - \frac{\sqrt{2^7}}{3} + \frac{3\sqrt{3}}{4} - \sqrt{\frac{2^3}{3^2}} = 2^2\sqrt{3} - \frac{2^3\sqrt{2}}{3} + \frac{3\sqrt{3}}{4} - \frac{2}{3}\sqrt{2} =$$

$$= 4\sqrt{3} - \frac{8}{3}\sqrt{2} + \frac{3}{4}\sqrt{3} - \frac{2}{3}\sqrt{2} = \frac{19}{4}\sqrt{3} - \frac{10}{3}\sqrt{2}$$

l) $\sqrt{512} + \sqrt{648} - \sqrt{\frac{128}{81}} = \sqrt{2^9} + \sqrt{2^3 \cdot 3^4} - \sqrt{\frac{2^7}{3^4}} = 2^4 \sqrt{2} + 2 \cdot 3^2 \sqrt{2} - \frac{2^3}{3^2} \sqrt{2} = 16\sqrt{2} + 18\sqrt{2} - \frac{8}{9}\sqrt{2} =$
 $= \frac{298}{9}\sqrt{2}$

m) $\sqrt{75} - \frac{\sqrt{18}}{3} + \frac{3\sqrt{12}}{4} - \sqrt{\frac{2}{25}} = \sqrt{3 \cdot 5^2} - \frac{\sqrt{3^2 \cdot 2}}{3} + \frac{3\sqrt{2^2 \cdot 3}}{4} - \sqrt{\frac{2}{5^2}} = 5\sqrt{3} - \frac{3\sqrt{2}}{3} + \frac{3 \cdot 2\sqrt{3}}{4} - \frac{1}{5}\sqrt{2} =$
 $= 5\sqrt{3} - \sqrt{2} + \frac{3}{2}\sqrt{3} - \frac{1}{5}\sqrt{2} = \frac{13}{2}\sqrt{3} - \frac{6}{5}\sqrt{2}$

n) $\frac{1}{3}\sqrt{32} - 3\sqrt{27} - 5\sqrt{12} + \sqrt{50} = \frac{1}{3}\sqrt{2^5} - 3\sqrt{3^3} - 5\sqrt{2^2 \cdot 3} + \sqrt{2 \cdot 5^2} = \frac{1}{3} \cdot 2^2 \sqrt{2} - 3 \cdot 3\sqrt{3} - 5 \cdot 2\sqrt{3} + 5\sqrt{2} =$
 $= \frac{4}{3}\sqrt{2} - 9\sqrt{3} - 10\sqrt{3} + 5\sqrt{2} = \frac{19}{3}\sqrt{2} - 19\sqrt{3}$

o) $\sqrt{40} - 5\sqrt{90} - 5\sqrt{10} = \sqrt{2^3 \cdot 5} - 5\sqrt{2 \cdot 3^2 \cdot 5} - 5\sqrt{2 \cdot 5} = 2\sqrt{2 \cdot 5} - 5 \cdot 3\sqrt{2 \cdot 5} - 5\sqrt{2 \cdot 5} =$
 $= 2\sqrt{10} - 15\sqrt{10} - 5\sqrt{10} = -18\sqrt{10}$

p) $\sqrt{108x^3} - \sqrt{300x^5} + \sqrt{\frac{36x}{363}} = \sqrt{2^2 \cdot 3^3 \cdot x^3} - \sqrt{3 \cdot 2^2 \cdot 5^2 \cdot x^5} + \sqrt{\frac{2^2 \cdot 3^2 x}{3 \cdot 11^2}} \stackrel{\text{simplificamos el radicando del tercer radical}}{=} =$
 $= \sqrt{2^2 \cdot 3^3 \cdot x^3} - \sqrt{3 \cdot 2^2 \cdot 5^2 \cdot x^5} + \sqrt{\frac{2^2 \cdot 3 \cdot x}{11^2}} = 2 \cdot 3 \cdot x \cdot \sqrt{3x} - 2 \cdot 5 \cdot x^2 \sqrt{3x} + \frac{2}{11}\sqrt{3x} =$
 $= \left(6x - 10x^2 + \frac{2}{11}\right)\sqrt{3x}$

q) $\sqrt[6]{8} + \sqrt[4]{4} - 7\sqrt{72} = \sqrt[6]{2^3} + \sqrt[4]{2^2} - 7\sqrt{2^3 \cdot 3^2} \stackrel{\text{simplificar}}{=} = \sqrt{2} + \sqrt{2} - 7\sqrt{2^3 \cdot 3^2} \stackrel{\text{extraer factores}}{=} = \sqrt{2} + \sqrt{2} - 7 \cdot 2 \cdot 3\sqrt{2} =$
 $= \sqrt{2} + \sqrt{2} - 42\sqrt{2} = -40\sqrt{2}$

r) $\sqrt[6]{6561a^2} + \sqrt[3]{3993a} - \sqrt[3]{3a^4} = \sqrt[6]{3^8 a^2} + \sqrt[3]{3 \cdot 11^3 a} - \sqrt[3]{3a^4} \stackrel{\text{simplificar}}{=} = \sqrt[3]{3^4 a} + \sqrt[3]{3 \cdot 11^3 a} - \sqrt[3]{3a^4} \stackrel{\text{extraer factores}}{=} =$
 $= 3\sqrt[3]{3a} + 11\sqrt[3]{3a} - a\sqrt[3]{3a} = (14 - a)\sqrt[3]{3a}$

s) $\sqrt[3]{0,04} \cdot \sqrt{0,2} = \sqrt[3]{\frac{4}{100}} \cdot \sqrt{\frac{2}{10}} = \sqrt[3]{\frac{1}{25}} \cdot \sqrt{\frac{1}{5}} = \sqrt[3]{\frac{1}{5^2}} \cdot \sqrt{\frac{1}{5}} = \sqrt[6]{\frac{1}{5^4}} \cdot \sqrt[6]{\frac{1}{5^3}} = \sqrt[6]{\frac{1}{5^7}} = \frac{1}{\sqrt[6]{5^7}} = \frac{1}{5\sqrt[6]{5}}$

t) $(\sqrt{2})^3 \cdot \frac{\sqrt[3]{2}}{\sqrt{6}} = \frac{\sqrt{2^3} \cdot \sqrt[3]{2}}{\sqrt{2 \cdot 3}} = \sqrt[6]{\frac{2^9 \cdot 2^2}{2^3 \cdot 3^3}} = \sqrt[6]{\frac{2^{11}}{2^3 \cdot 3^3}} = \sqrt[6]{\frac{2^8}{3^3}} = 2\sqrt[6]{\frac{2^2}{3^3}} = 2\sqrt[6]{\frac{4}{27}}$

u) $\sqrt[4]{4} \cdot \sqrt[6]{8} \cdot \sqrt[8]{81} = \sqrt[4]{2^2} \cdot \sqrt[6]{2^3} \cdot \sqrt[8]{3^4} \stackrel{\text{simplificar}}{=} = \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{3} = (\sqrt{2})^2 \cdot \sqrt{3} = 2\sqrt{3}$

$$v) \frac{\sqrt{2} \cdot \sqrt[3]{4}}{\sqrt[3]{\sqrt{2}}} = \frac{\sqrt{2} \cdot \sqrt[6]{2^2}}{\sqrt[6]{2}} = \sqrt[6]{\frac{2^3 \cdot 2^2}{2}} = \sqrt[6]{2^4} = \sqrt[3]{2^2} = \sqrt[3]{4}$$

$$w) -\sqrt[3]{-16} - \sqrt[3]{-54} - \sqrt[3]{-250} = \sqrt[3]{16} + \sqrt[3]{54} + \sqrt[3]{250} = \sqrt[3]{2^4} + \sqrt[3]{2 \cdot 3^3} + \sqrt[3]{2 \cdot 5^3} = 2\sqrt[3]{2} + 3\sqrt[3]{2} + 5\sqrt[3]{2} = 10\sqrt[3]{2}$$

8. Calcula:

$$a) (2\sqrt{3} + \sqrt{5})^2 = (2\sqrt{3})^2 + 2 \cdot 2\sqrt{3} \cdot \sqrt{5} + (\sqrt{5})^2 = 12 + 4\sqrt{15} + 5 = 17 + 4\sqrt{15}$$

$$b) 2\sqrt{6} \cdot (2\sqrt{5} - \sqrt{2})^2 = 2\sqrt{6} \cdot [(2\sqrt{5})^2 - 2 \cdot 2\sqrt{5} \cdot \sqrt{2} + (\sqrt{2})^2] = 2\sqrt{6} \cdot (20 - 4\sqrt{10} + 2) = \\ = 2\sqrt{6} \cdot (22 - 4\sqrt{10}) = 44\sqrt{6} - 8\sqrt{60} = 44\sqrt{6} - 8\sqrt{2^2 \cdot 3 \cdot 5} = 44\sqrt{6} - 8 \cdot 2\sqrt{2} \cdot \sqrt{3 \cdot 5} = 44\sqrt{6} - 16\sqrt{15}$$

$$c) (\sqrt{2} + 1)^2 \cdot \sqrt{3} = [(\sqrt{2})^2 + 2 \cdot \sqrt{2} \cdot 1 + 1^2] \cdot \sqrt{3} = (2 + 2\sqrt{2} + 1) \cdot \sqrt{3} = (3 + 2\sqrt{2}) \cdot \sqrt{3} = 3\sqrt{3} + 2\sqrt{6}$$

$$d) (2\sqrt{2} - \sqrt{3}) \cdot (\sqrt{3} + 3) = 2\sqrt{6} + 6\sqrt{2} - 3 - 3\sqrt{3}$$

$$e) [(\sqrt{2} - 1)^2 - 1] \cdot \sqrt{2} = [(\sqrt{2})^2 - 2 \cdot \sqrt{2} \cdot 1 + 1^2 - 1] \cdot \sqrt{2} = [2 - 2\sqrt{2} + 1 - 1] \cdot \sqrt{2} = (2 - 2\sqrt{2}) \cdot \sqrt{2} = 2\sqrt{2} - 4$$

$$f) (1 + \sqrt{2}) \cdot (1 - \sqrt{2}) = 1^2 - (\sqrt{2})^2 = 1 - 2 = -1$$

$$g) (2\sqrt{3} - \sqrt{6})^2 = (2\sqrt{3})^2 - 2 \cdot 2\sqrt{3} \cdot \sqrt{6} + (\sqrt{6})^2 = 12 - 4\sqrt{18} + 6 = 18 - 4\sqrt{3^2 \cdot 2} = 18 - 4 \cdot 3\sqrt{2} = 18 - 12\sqrt{2}$$

$$h) (2\sqrt{5} - 3\sqrt{10})^2 = (2\sqrt{5})^2 - 2 \cdot 2\sqrt{5} \cdot 3\sqrt{10} + (3\sqrt{10})^2 = 4 \cdot 5 - 12\sqrt{50} + 9 \cdot 10 = 20 - 12\sqrt{50} + 90 = \\ = 110 - 12\sqrt{5^2 \cdot 2} = 110 - 12 \cdot 5\sqrt{2} = 110 - 60\sqrt{2}$$

$$i) (1 + 3\sqrt{3}) \cdot (2\sqrt{3} + 4) = 2\sqrt{3} + 4 + 6 \cdot 3 + 12\sqrt{3} = 2\sqrt{3} + 4 + 18 + 12\sqrt{3} = 14\sqrt{3} + 22$$

$$j) (\sqrt{7} - \sqrt{18})^2 + 3\sqrt{56} = [(\sqrt{7})^2 - 2 \cdot \sqrt{7} \cdot \sqrt{18} + (\sqrt{18})^2] + 3\sqrt{56} = [7 - 2\sqrt{7 \cdot 2 \cdot 3^2} + 18] + 3\sqrt{2^3 \cdot 7} = \\ = [25 - 2 \cdot 3\sqrt{7 \cdot 2}] + 3 \cdot 2\sqrt{2 \cdot 7} = (25 - 6\sqrt{14}) + 6\sqrt{14} = 25$$

$$k) (3\sqrt{3} - 2\sqrt{8}) \cdot (4 + 3\sqrt{6}) = 12\sqrt{3} + 9\sqrt{18} - 8\sqrt{8} - 6\sqrt{48} = 12\sqrt{3} + 9\sqrt{3^2 \cdot 2} - 8\sqrt{2^3} - 6\sqrt{2^4 \cdot 3} = \\ = 12\sqrt{3} + 9 \cdot 3\sqrt{2} - 8 \cdot 2\sqrt{2} - 6 \cdot 2^2 \sqrt{3} = 12\sqrt{3} + 27\sqrt{2} - 16\sqrt{2} - 24\sqrt{3} = 11\sqrt{2} - 12\sqrt{3}$$

$$l) (1 + 3\sqrt{3}) \cdot (2\sqrt{3} + 4) = 2\sqrt{3} + 4 + 6 \cdot 3 + 12\sqrt{3} = 2\sqrt{3} + 4 + 18 + 12\sqrt{3} = 14\sqrt{3} + 22$$

$$m) (3 + 4\sqrt{2}) \cdot (3 - 4\sqrt{2}) = (3)^2 - (4\sqrt{2})^2 = 9 - 16 \cdot 2 = 9 - 32 = -23$$

$$n) (2 - 3\sqrt{2})^2 = 2^2 - 2 \cdot 2 \cdot 3\sqrt{2} + (3\sqrt{2})^2 = 4 - 12\sqrt{2} + 9 \cdot 2 = 4 - 12\sqrt{2} + 18 = 22 - 12\sqrt{2}$$

$$o) (3 - 2\sqrt{5}) \cdot (4 + 3\sqrt{5}) = 12 + 9\sqrt{5} - 8\sqrt{5} - 6 \cdot 5 = 12 + 9\sqrt{5} - 8\sqrt{5} - 30 = \sqrt{5} - 18$$

$$p) (3\sqrt{5} + 4\sqrt{2}) \cdot (3\sqrt{5} - 4\sqrt{2}) = (3\sqrt{5})^2 - (4\sqrt{2})^2 = 9 \cdot 5 - 16 \cdot 2 = 45 - 32 = 13$$

9. Calcula:

$$a) \sqrt{4 \cdot \sqrt[3]{8} \cdot \sqrt[4]{256}} = \sqrt{2^2 \cdot \sqrt[3]{2^3} \cdot \sqrt[4]{2^8}} = \sqrt{2^2 \cdot \sqrt[3]{2^3} \cdot 2^2} = \sqrt{2^2 \cdot \sqrt[3]{2^5}} = \sqrt{\sqrt[3]{2^6} \cdot 2^5} = \sqrt[6]{2^{11}} = 2 \sqrt[6]{2^5} = 2 \sqrt[6]{32}$$

$$b) \sqrt{9 \cdot \sqrt{3} \cdot \sqrt{27}} = \sqrt{3^2 \cdot \sqrt{3} \cdot \sqrt{3^3}} = \sqrt{3^2 \cdot \sqrt{\sqrt{3^2} \cdot 3^3}} = \sqrt{3^2 \cdot \sqrt[4]{3^5}} = \sqrt{\sqrt[4]{3^8} \cdot 3^5} = \sqrt[8]{3^{13}} = 3 \sqrt[8]{3^5} = 3 \sqrt[8]{243}$$

$$c) \sqrt{3 \cdot \sqrt{3} \cdot \sqrt{3}} = \sqrt{3 \cdot \sqrt{\sqrt{3^2} \cdot 3}} = \sqrt{3 \cdot \sqrt[4]{3^3}} = \sqrt{\sqrt[4]{3^4} \cdot 3^3} = \sqrt[8]{3^7} = \sqrt[8]{2187}$$

$$d) \sqrt{2 \cdot \sqrt[3]{8} \cdot \sqrt{32}} = \sqrt{2 \cdot \sqrt[3]{2^3} \cdot \sqrt{2^5}} = \sqrt{2 \cdot \sqrt[3]{\sqrt{2^6} \cdot 2^5}} = \sqrt{2 \cdot \sqrt[6]{2^{11}}} = \sqrt{\sqrt[6]{2^6} \cdot 2^{11}} = \sqrt[12]{2^{17}} = 2 \sqrt[12]{2^5} = 2 \sqrt[12]{32}$$

$$e) \sqrt{3 \cdot \sqrt[3]{9} \cdot \sqrt{27}} = \sqrt{3 \cdot \sqrt[3]{3^2} \cdot \sqrt{3^3}} = \sqrt{3 \cdot \sqrt[3]{\sqrt{3^4} \cdot 3^3}} = \sqrt{3 \cdot \sqrt[6]{3^7}} = \sqrt{\sqrt[6]{3^6} \cdot 3^7} = \sqrt[12]{3^{13}} = 3 \sqrt[12]{3}$$

$$f) \sqrt{4 \cdot \sqrt[3]{8} \cdot \sqrt[4]{256}} \cdot \sqrt[4]{2 \cdot \sqrt[3]{16}} = \sqrt{2^2 \cdot \sqrt[3]{2^3} \cdot \sqrt[4]{2^8}} \cdot \sqrt[4]{2 \cdot \sqrt[3]{2^4}} = \sqrt{2^2 \cdot 2 \cdot 2^2} \cdot \sqrt[4]{\sqrt[3]{2^3} \cdot 2^4} = \sqrt{2^5} \cdot \sqrt[12]{2^7} = \sqrt[12]{2^{30}} \cdot \sqrt[12]{2^7} = \sqrt[12]{2^{37}} = 2^3 \cdot \sqrt[12]{2} = 8 \sqrt[12]{2}$$

$$g) \left(\sqrt{4 \cdot \sqrt[3]{16}}\right)^6 = \left(\sqrt{2^2 \cdot \sqrt[3]{2^4}}\right)^6 = \left(\sqrt{\sqrt[3]{2^6} \cdot 2^4}\right)^6 = \left(\sqrt[6]{2^{10}}\right)^6 = 2^{10} = 1024$$

$$h) \sqrt[5]{4 \cdot \sqrt[3]{2} \cdot \sqrt{128}} = \sqrt[5]{2^2 \cdot \sqrt[3]{2^2} \cdot \sqrt{2^7}} = \sqrt[5]{2^2 \cdot \sqrt[6]{2^9}} = \sqrt[5]{\sqrt[6]{2^{12}} \cdot 2^9} = \sqrt[30]{2^{21}} = \sqrt[10]{2^7} = \sqrt[10]{128}$$

$$i) 3 \cdot \sqrt{\frac{4x+8}{25}} - 2 \cdot \sqrt{9x+18} = 3 \cdot \sqrt{\frac{4}{25} \cdot (x+2)} - 2 \cdot \sqrt{9 \cdot (x+2)} = 3 \cdot \frac{2}{5} \sqrt{x+2} - 2 \cdot 3 \sqrt{x+2} = \frac{6}{5} \sqrt{x+2} - 6 \sqrt{x+2} = \left(\frac{6}{5} - 6\right) \sqrt{x+2} = -\frac{24}{5} \sqrt{x+2}$$

10. Racionaliza los denominadores de las siguientes expresiones:

$$a) \frac{3}{\sqrt{6}} = \frac{3 \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$$

$$b) \frac{3-\sqrt{3}}{\sqrt{6}} = \frac{(3-\sqrt{3}) \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{3\sqrt{6} - \sqrt{18}}{6} = \frac{3\sqrt{6} - \sqrt{2 \cdot 3^2}}{6} = \frac{3\sqrt{6} - 3\sqrt{2}}{6} = \frac{\sqrt{6} - \sqrt{2}}{2}$$

$$c) \frac{6}{\sqrt[3]{32}} = \frac{6}{\sqrt[3]{2^5}} = \frac{6 \cdot \sqrt[3]{2^4}}{\sqrt[3]{2^5} \cdot \sqrt[3]{2^4}} = \frac{6 \cdot \sqrt[3]{16}}{\sqrt[3]{2^9}} = \frac{6 \cdot \sqrt[3]{16}}{2} = 3 \sqrt[3]{16}$$

$$d) \frac{1+\sqrt{3}}{\sqrt{3}-1} = \frac{(1+\sqrt{3}) \cdot (\sqrt{3}+1)}{(\sqrt{3}-1) \cdot (\sqrt{3}+1)} = \frac{\sqrt{3}+1+3+\sqrt{3}}{(\sqrt{3})^2 - (1)^2} = \frac{4+2\sqrt{3}}{3-1} = \frac{4+2\sqrt{3}}{2} = 2 + \sqrt{3}$$

$$e) \frac{5-\sqrt{10}}{\sqrt{5}} = \frac{(5-\sqrt{10}) \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{5\sqrt{5} - \sqrt{50}}{5} = \frac{5\sqrt{5} - \sqrt{2 \cdot 5^2}}{5} = \frac{5\sqrt{5} - 5\sqrt{2}}{5} = \sqrt{5} - \sqrt{2}$$

$$f) \frac{5+\sqrt{3}}{\sqrt{3}-5} = \frac{(5+\sqrt{3}) \cdot (\sqrt{3}+5)}{(\sqrt{3}-5) \cdot (\sqrt{3}+5)} = \frac{5\sqrt{3} + 25 + 3 + 5\sqrt{3}}{(\sqrt{3})^2 - (5)^2} = \frac{28+10\sqrt{3}}{3-25} = \frac{28+10\sqrt{3}}{-22} = \frac{-14-5\sqrt{3}}{11}$$

$$g) \frac{2}{2\sqrt{3}-\sqrt{2}} = \frac{2 \cdot (2\sqrt{3}+\sqrt{2})}{(2\sqrt{3}-\sqrt{2}) \cdot (2\sqrt{3}+\sqrt{2})} = \frac{4\sqrt{3}+2\sqrt{2}}{(2\sqrt{3})^2-(\sqrt{2})^2} = \frac{4\sqrt{3}+2\sqrt{2}}{4 \cdot 3-2} = \frac{4\sqrt{3}+2\sqrt{2}}{10} =$$

$$= \frac{2\sqrt{3}+\sqrt{2}}{5}$$

Simplificar (:2)

$$h) \frac{1+\sqrt[3]{2}}{\sqrt[3]{4}} = \frac{1+\sqrt[3]{2}}{\sqrt[3]{2^2}} = \frac{(1+\sqrt[3]{2}) \cdot \sqrt[3]{2}}{\sqrt[3]{2^2} \cdot \sqrt[3]{2}} = \frac{\sqrt[3]{2} + \sqrt[3]{4}}{\sqrt[3]{2^3}} = \frac{\sqrt[3]{2} + \sqrt[3]{4}}{2}$$

$$i) \frac{1-2\sqrt{2}}{3\sqrt{8}-1} = \frac{(1-2\sqrt{2}) \cdot (3\sqrt{8}+1)}{(3\sqrt{8}-1) \cdot (3\sqrt{8}+1)} = \frac{3\sqrt{8}+1-6\sqrt{16}-2\sqrt{2}}{(3\sqrt{8})^2-(1)^2} = \frac{3\sqrt{2^3}+1-6 \cdot 4-2\sqrt{2}}{9 \cdot 8-1} =$$

$$= \frac{3 \cdot 2\sqrt{2}+1-24-2\sqrt{2}}{72-1} = \frac{6\sqrt{2}-23-2\sqrt{2}}{71} = \frac{4\sqrt{2}-23}{71}$$

$$j) \frac{3\sqrt{6}-2\sqrt{3}}{2\sqrt{6}-3\sqrt{3}} = \frac{(3\sqrt{6}-2\sqrt{3}) \cdot (2\sqrt{6}+3\sqrt{3})}{(2\sqrt{6}-3\sqrt{3}) \cdot (2\sqrt{6}+3\sqrt{3})} = \frac{6 \cdot 6+9\sqrt{18}-4\sqrt{18}-6 \cdot 3}{(2\sqrt{6})^2-(3\sqrt{3})^2} = \frac{36+5\sqrt{18}-18}{4 \cdot 6-9 \cdot 3} =$$

$$= \frac{18+5\sqrt{2 \cdot 3^2}}{24-27} = \frac{18+5 \cdot 3\sqrt{2}}{-3} = \frac{18+15\sqrt{2}}{-3} = -6-5\sqrt{2}$$

Simplificar (:-3)

$$k) \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{(\sqrt{a}+\sqrt{b}) \cdot (\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b}) \cdot (\sqrt{a}+\sqrt{b})} = \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b}) \cdot (\sqrt{a}+\sqrt{b})} = \frac{(\sqrt{a})^2+(\sqrt{b})^2+2\sqrt{a}\sqrt{b}}{(\sqrt{a})^2-(\sqrt{b})^2} = \frac{a+b+2\sqrt{ab}}{a-b}$$

$$l) \frac{\sqrt{3}+1}{\sqrt{2}} = \frac{(\sqrt{3}+1) \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$$

$$m) \frac{1-\sqrt{5}}{2\sqrt{5}+1} = \frac{(1-\sqrt{5}) \cdot (2\sqrt{5}-1)}{(2\sqrt{5}+1) \cdot (2\sqrt{5}-1)} = \frac{2\sqrt{5}-1-2 \cdot 5+\sqrt{5}}{(2\sqrt{5})^2-(1)^2} = \frac{2\sqrt{5}-1-10+\sqrt{5}}{4 \cdot 5-1} = \frac{3\sqrt{5}-11}{19}$$

$$n) \frac{7}{\sqrt{3} \cdot \sqrt[3]{2}} = \frac{7\sqrt{3}}{\sqrt{3} \cdot \sqrt{3} \cdot \sqrt[3]{2}} = \frac{7\sqrt{3}}{3 \cdot \sqrt[3]{2}} = \frac{7\sqrt{3} \cdot \sqrt[3]{2^2}}{3 \cdot \sqrt[3]{2} \cdot \sqrt[3]{2^2}} = \frac{7\sqrt[6]{3^3} \cdot \sqrt[6]{2^4}}{3\sqrt[3]{2^3}} = \frac{7\sqrt[6]{3^3 \cdot 2^4}}{3 \cdot 2} = \frac{7\sqrt[6]{432}}{6}$$

$$\tilde{n}) \frac{\sqrt{3}}{\sqrt{2\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{\sqrt{2^2 \cdot 3}}} = \frac{\sqrt{3}}{\sqrt[4]{2^2 \cdot 3}} = \frac{\sqrt{3} \cdot \sqrt[4]{2^2 \cdot 3^3}}{\sqrt[4]{2^2 \cdot 3} \cdot \sqrt[4]{2^2 \cdot 3^3}} = \frac{\sqrt[4]{3^2} \cdot \sqrt[4]{2^2 \cdot 3^3}}{\sqrt[4]{2^4 \cdot 3^4}} = \frac{\sqrt[4]{2^2 \cdot 3^5}}{2 \cdot 3} = \frac{3\sqrt[4]{2^2 \cdot 3}}{2 \cdot 3} = \frac{\sqrt[4]{12}}{2}$$

$$o) \frac{2-\sqrt{2}}{2\sqrt{3}} = \frac{(2-\sqrt{2}) \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{3}-\sqrt{6}}{2 \cdot 3} = \frac{2\sqrt{3}-\sqrt{6}}{6}$$

$$p) \frac{\sqrt[3]{6}-2}{\sqrt[3]{9}} = \frac{\sqrt[3]{6}-2}{\sqrt[3]{3^2}} = \frac{(\sqrt[3]{6}-2) \cdot \sqrt[3]{3}}{\sqrt[3]{3^2} \cdot \sqrt[3]{3}} = \frac{\sqrt[3]{18}-2\sqrt[3]{3}}{\sqrt[3]{3^3}} = \frac{\sqrt[3]{18}-2\sqrt[3]{3}}{3}$$

$$q) \frac{3\sqrt{3}-2\sqrt{2}}{2\sqrt{3}+3\sqrt{2}} = \frac{(3\sqrt{3}-2\sqrt{2}) \cdot (2\sqrt{3}-3\sqrt{2})}{(2\sqrt{3}+3\sqrt{2}) \cdot (2\sqrt{3}-3\sqrt{2})} = \frac{6 \cdot 3-9\sqrt{6}-4\sqrt{6}+6 \cdot 2}{(2\sqrt{3})^2-(3\sqrt{2})^2} = \frac{18-13\sqrt{6}+12}{4 \cdot 3-9 \cdot 2} =$$

$$= \frac{30-13\sqrt{6}}{12-18} = \frac{30-13\sqrt{6}}{-6} = -\frac{30-13\sqrt{6}}{6}$$

11. Calcula y simplifica:

$$a) \frac{3^{\frac{3}{4}} \cdot 9^{\frac{3}{2}}}{(\sqrt{3})^{-3} \cdot \sqrt[3]{81}} = \frac{3^{\frac{3}{4}} \cdot (3^2)^{\frac{3}{2}}}{\left(3^{\frac{1}{2}}\right)^{-3} \cdot \sqrt[3]{3^4}} = \frac{3^{\frac{3}{4}} \cdot 3^3}{3^{\frac{3}{2}} \cdot 3^{\frac{4}{3}}} = \frac{3^{\frac{3}{4}+3}}{3^{\frac{3}{2}+\frac{4}{3}}} = \frac{3^{\frac{9}{4}}}{3^{\frac{17}{6}}} = 3^{\frac{9}{4}-\frac{17}{6}} = 3^{\frac{9+1}{6}} = 3^{\frac{29}{6}}$$

$$b) (\sqrt{5})^3 \cdot \frac{\sqrt[3]{20}}{\sqrt{10}} = (\sqrt{5})^3 \cdot \frac{\sqrt{5^3} \cdot \sqrt[3]{2^2 \cdot 5}}{\sqrt{2 \cdot 5}} = \frac{\sqrt[6]{5^9} \cdot \sqrt[6]{2^4 \cdot 5^2}}{\sqrt[6]{2^3 \cdot 5^3}} = \frac{\sqrt[6]{2^4 \cdot 5^{11}}}{\sqrt[6]{2^3 \cdot 5^3}} = \sqrt[6]{2 \cdot 5^8} = 5 \cdot \sqrt[6]{2 \cdot 5^2} = 5 \cdot \sqrt[6]{50}$$

$$c) \frac{5}{2}\sqrt{32} + 7\sqrt{27} - \frac{9}{2}\sqrt{3} + \frac{5}{3}\sqrt{18} = \frac{5}{2}\sqrt{2^5} + 7\sqrt{3^3} - \frac{9}{2}\sqrt{3} + \frac{5}{3}\sqrt{2 \cdot 3^2} =$$

$$= \frac{5}{2} \cdot 2^2 \sqrt{2} + 7 \cdot 3\sqrt{3} - \frac{9}{2}\sqrt{3} + \frac{5}{3} \cdot 3\sqrt{2} = 10\sqrt{2} + 21\sqrt{3} - \frac{9}{2}\sqrt{3} + 5\sqrt{2} = 15\sqrt{2} + \frac{33}{2}\sqrt{3}$$

$$d) \frac{\sqrt{3} \cdot \sqrt[3]{9}}{\sqrt[3]{\sqrt{3}}} = \frac{\sqrt{3} \cdot \sqrt[6]{3^2}}{\sqrt[6]{3}} = \frac{\sqrt[6]{3^3} \cdot \sqrt[6]{3^2}}{\sqrt[6]{3}} = \frac{\sqrt[6]{3^5}}{\sqrt[6]{3}} = \sqrt[6]{3^4} \stackrel{\text{Simplificar } (:2)}{=} \sqrt[3]{3^2} = \sqrt[3]{9}$$

$$e) 2\sqrt{5} \cdot (\sqrt{20} - 8\sqrt{45}) - (\sqrt{3} + \sqrt{5}) \cdot (\sqrt{3} - \sqrt{5}) =$$

$$= 2\sqrt{5} \cdot (\sqrt{2^2 \cdot 5} - 8\sqrt{3^2 \cdot 5}) - (\sqrt{3} + \sqrt{5}) \cdot (\sqrt{3} - \sqrt{5}) = 2\sqrt{5} \cdot (2\sqrt{5} - 24\sqrt{5}) - (\sqrt{3} + \sqrt{5}) \cdot (\sqrt{3} - \sqrt{5}) =$$

$$= 2\sqrt{5} \cdot (-22\sqrt{5}) - (\sqrt{3} + \sqrt{5}) \cdot (\sqrt{3} - \sqrt{5}) = -44(\sqrt{5})^2 - [(\sqrt{3})^2 - (\sqrt{5})^2] = -44 \cdot 5 - (3 - 5) = -220 + 2 = -218$$

$$(\sqrt{a-b} + \sqrt{a+b}) \cdot (\sqrt{a-b} - \sqrt{a+b}) = (\sqrt{a-b})^2 - (\sqrt{a+b})^2 = (a-b) - (a+b) = a-b-a-b = -2b$$

$$f) \frac{\sqrt{3} + \sqrt{2}}{\sqrt{5} - \sqrt{6}} + \frac{\sqrt{3} - \sqrt{2}}{\sqrt{5} + \sqrt{6}} \stackrel{(*)}{=} (-\sqrt{15} - 3\sqrt{2} - \sqrt{10} - 2\sqrt{3}) + (-\sqrt{15} + 3\sqrt{2} + \sqrt{10} - 2\sqrt{3}) = -2\sqrt{15} - 4\sqrt{3}$$

$$(*) \frac{\sqrt{3} + \sqrt{2}}{\sqrt{5} - \sqrt{6}} = \frac{(\sqrt{3} + \sqrt{2})(\sqrt{5} + \sqrt{6})}{(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{6})} = \frac{\sqrt{15} + \sqrt{18} + \sqrt{10} + \sqrt{12}}{(\sqrt{5})^2 - (\sqrt{6})^2} = \frac{\sqrt{15} + \sqrt{3^2 \cdot 2} + \sqrt{10} + \sqrt{2^2 \cdot 3}}{5 - 6} =$$

$$= \frac{\sqrt{15} + 3\sqrt{2} + \sqrt{10} + 2\sqrt{3}}{-1} = -\sqrt{15} - 3\sqrt{2} - \sqrt{10} - 2\sqrt{3}$$

$$(*) \frac{\sqrt{3} - \sqrt{2}}{\sqrt{5} + \sqrt{6}} = \frac{(\sqrt{3} - \sqrt{2})(\sqrt{5} - \sqrt{6})}{(\sqrt{5} + \sqrt{6})(\sqrt{5} - \sqrt{6})} = \frac{\sqrt{15} - \sqrt{18} - \sqrt{10} + \sqrt{12}}{(\sqrt{5})^2 - (\sqrt{6})^2} = \frac{\sqrt{15} - \sqrt{3^2 \cdot 2} - \sqrt{10} + \sqrt{2^2 \cdot 3}}{5 - 6} =$$

$$= \frac{\sqrt{15} - 3\sqrt{2} - \sqrt{10} + 2\sqrt{3}}{-1} = -\sqrt{15} + 3\sqrt{2} + \sqrt{10} - 2\sqrt{3}$$

$$g) \frac{\sqrt{5} + \sqrt{3}}{3\sqrt{3} - 5\sqrt{5}} + \frac{\sqrt{5} - \sqrt{3}}{3\sqrt{3} + 5\sqrt{5}} = \frac{-4\sqrt{15} - 17}{49} + \frac{-4\sqrt{15} + 17}{49} = \frac{-4\sqrt{15} - 17 - 4\sqrt{15} + 17}{49} = \frac{-8\sqrt{15}}{49}$$

$$(*) \frac{\sqrt{5} + \sqrt{3}}{3\sqrt{3} - 5\sqrt{5}} = \frac{(\sqrt{5} + \sqrt{3})(3\sqrt{3} + 5\sqrt{5})}{(3\sqrt{3} - 5\sqrt{5})(3\sqrt{3} + 5\sqrt{5})} = \frac{3\sqrt{15} + 5(\sqrt{5})^2 + 3(\sqrt{3})^2 + 5\sqrt{15}}{(3\sqrt{3})^2 - (5\sqrt{5})^2} = \frac{3\sqrt{15} + 25 + 9 + 5\sqrt{15}}{9 \cdot 3 - 25 \cdot 5} =$$

$$= \frac{8\sqrt{15} + 34}{27 - 125} = \frac{8\sqrt{15} + 34}{-98} \stackrel{(:-2)}{=} \frac{-4\sqrt{15} - 17}{49}$$

$$\begin{aligned}
 (*) \frac{\sqrt{5}-\sqrt{3}}{3\sqrt{3}+5\sqrt{5}} &= \frac{(\sqrt{5}-\sqrt{3})(3\sqrt{3}-5\sqrt{5})}{(3\sqrt{3}+5\sqrt{5})(3\sqrt{3}-5\sqrt{5})} = \frac{3\sqrt{15}-5(\sqrt{5})^2-3(\sqrt{3})^2+5\sqrt{15}}{(3\sqrt{3})^2-(5\sqrt{5})^2} = \frac{3\sqrt{15}-25-9+5\sqrt{15}}{9\cdot 3-25\cdot 5} = \\
 &= \frac{8\sqrt{15}-34}{27-125} = \frac{8\sqrt{15}-34}{-98} \stackrel{(:-2)}{=} \frac{-4\sqrt{15}+17}{49}
 \end{aligned}$$

$$\text{h) } \frac{\sqrt{3}-\sqrt{2}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}-3\sqrt{2}} \stackrel{(*)}{=} \frac{5\sqrt{6}-12}{6} + \frac{-5\sqrt{6}-12}{6} = \frac{5\sqrt{6}-12-5\sqrt{6}-12}{6} = \frac{-24}{6} = -4$$

$$\begin{aligned}
 (*) \frac{\sqrt{3}-\sqrt{2}}{3\sqrt{2}+2\sqrt{3}} &= \frac{(\sqrt{3}-\sqrt{2})(3\sqrt{2}-2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})(3\sqrt{2}-2\sqrt{3})} = \frac{3\sqrt{6}-2(\sqrt{3})^2-3(\sqrt{2})^2+2\sqrt{6}}{(3\sqrt{2})^2-(2\sqrt{3})^2} = \\
 &= \frac{3\sqrt{6}-2\cdot 3-3\cdot 2+2\sqrt{6}}{9\cdot 2-4\cdot 3} = \frac{3\sqrt{6}-6-6+2\sqrt{6}}{18-12} = \frac{5\sqrt{6}-12}{6}
 \end{aligned}$$

$$\begin{aligned}
 (*) \frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}-3\sqrt{2}} &= \frac{(\sqrt{3}+\sqrt{2})(2\sqrt{3}+3\sqrt{2})}{(2\sqrt{3}-3\sqrt{2})(2\sqrt{3}+3\sqrt{2})} = \frac{2(\sqrt{3})^2+3\sqrt{6}+2\sqrt{6}+3(\sqrt{2})^2}{(2\sqrt{3})^2-(3\sqrt{2})^2} = \\
 &= \frac{2\cdot 3+3\sqrt{6}+2\sqrt{6}+3\cdot 2}{4\cdot 3-9\cdot 2} = \frac{6+5\sqrt{6}+6}{12-18} = \frac{5\sqrt{6}+12}{-6} = \frac{-5\sqrt{6}-12}{6}
 \end{aligned}$$

$$\text{i) } \frac{5\sqrt{3}+\sqrt{6}}{2\sqrt{6}} - \frac{\sqrt{2}+1}{5-2\sqrt{2}} + \frac{3\sqrt{2}}{1+\sqrt{8}} = \frac{5\sqrt{2}+2}{4} - \frac{7\sqrt{2}+9}{17} + \frac{-3\sqrt{2}+12}{7} =$$

$$\begin{aligned}
 &= \frac{119(5\sqrt{2}+2)-28(7\sqrt{2}+9)+68(-3\sqrt{2}+12)}{476} = \frac{595\sqrt{2}+238-196\sqrt{2}-252-204\sqrt{2}+816}{476} = \\
 &= \frac{195\sqrt{2}+802}{476}
 \end{aligned}$$

$$(*) \frac{5\sqrt{3}+\sqrt{6}}{2\sqrt{6}} = \frac{(5\sqrt{3}+\sqrt{6})\cdot\sqrt{6}}{2\sqrt{6}\cdot\sqrt{6}} = \frac{5\sqrt{18}+(\sqrt{6})^2}{2(\sqrt{6})^2} = \frac{5\sqrt{2\cdot 3^2}+6}{2\cdot 6} = \frac{5\cdot 3\sqrt{2}+6}{12} = \frac{15\sqrt{2}+6}{12} \stackrel{(:3)}{=} \frac{5\sqrt{2}+2}{4}$$

$$\begin{aligned}
 (*) \frac{\sqrt{2}+1}{5-2\sqrt{2}} &= \frac{(\sqrt{2}+1)(5+2\sqrt{2})}{(5-2\sqrt{2})(5+2\sqrt{2})} = \frac{5\sqrt{2}+2(\sqrt{2})^2+5+2\sqrt{2}}{(5)^2-(2\sqrt{2})^2} = \frac{5\sqrt{2}+2\cdot 2+5+2\sqrt{2}}{25-4\cdot 2} = \frac{7\sqrt{2}+4+5}{25-8} = \\
 &= \frac{7\sqrt{2}+9}{17}
 \end{aligned}$$

$$(*) \frac{3\sqrt{2}}{1+\sqrt{8}} = \frac{3\sqrt{2}(1-\sqrt{8})}{(1+\sqrt{8})(1-\sqrt{8})} = \frac{3\sqrt{2}-3\sqrt{16}}{(1)^2-(\sqrt{8})^2} = \frac{3\sqrt{2}-3\cdot 4}{1-8} = \frac{3\sqrt{2}-12}{-7} = \frac{-3\sqrt{2}+12}{7}$$