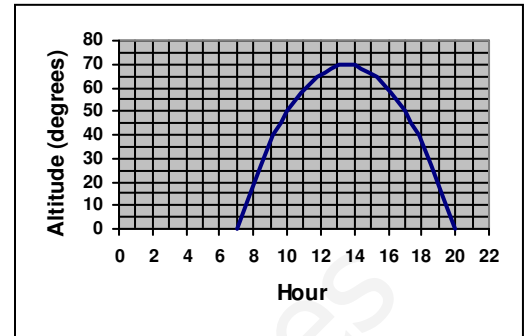


## 2<sup>nd</sup> term General Exam

1.- The following graph shows the altitude of the sun on the horizon, in degrees, on a certain day.

- What time is sunrise? And sunset?
- When is the altitude increasing? And decreasing?
- What time does the sun reach maximum height above the horizon?
- How many hours of daylight were there this day?



2.- Simplify:

a)  $\frac{x^3 - x}{2x^2 + 2x} =$

b)  $\frac{x^2 - 6x + 9}{3x^2 - 9x} =$

3.- Complete:

a)  $9x^2 - \quad + \quad = (\quad - 2)^2$

b)  $y^2 - 25x^2 = (\quad)(\quad)$

c)  $(3 + \quad)^2 = \quad + 12x^2 + \quad$

4.- Solve:  $2(x-1)^2 + 4x - (x+1)^2 - x^2 = 1 - 2x$

5.- Solve:  $\frac{2x}{3} - \frac{x-2}{5} = 1 - \frac{(x-3)^2}{15}$

6.- The measure of the unequal side of an isosceles triangle is 21 cm. If the perimeter is 87 cm, how long are the other sides?

7.- Bob is five times as old as Carlos and in 8 years' time he will be three times as old as him. How old is each one?

8.- Two cyclists leave at 9 in the morning from two towns (A and B) located at 150 km away from each other. The cyclist that leaves from A goes at 34 km/h and the one that leaves from B at 26 km/h. How far from A and B will they meet up? When will they meet up?

9.- A hotel has double rooms and single rooms. In total there are 58 rooms and 100 beds. How many rooms are there of each type?

10.- Solve the following system by graphing and by another method (substitution or addition):

$$\begin{cases} 3 + 4x = 2y \\ 2x - y = -1 \end{cases}$$

(1 point each question)

## SOLUTION

1.- The following graph shows the altitude of the sun on the horizon, in degrees, on a certain day.

c) What time is sunrise? And sunset?

Sunrise is at 7 h

Sunset is at 20 h

d) When is the altitude increasing? And decreasing?

The altitude is increasing from 7 to 13:30

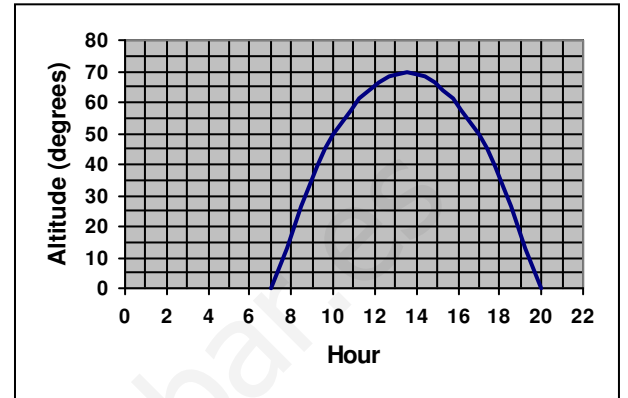
It is decreasing from 13:30 to 20

c) What time does the sun reach maximum height above the horizon?

At 13:30 approximately

d) How many hours of daylight were there this day?

$20 - 7 = 13$  hours of daylight



2.- Simplify:

$$a) \frac{x^3 - x}{2x^2 + 2x} = \frac{x(x^2 - 1)}{2x(x+1)} = \frac{x(x+1)(x-1)}{2x(x+1)} = \frac{x-1}{2}$$

$$b) \frac{x^2 - 6x + 9}{3x^2 - 9x} = \frac{(x-3)^2}{3x(x-3)} = \frac{x-3}{3x}$$

3.- Complete:

$$a) 9x^2 - 12x + 4 = (3x - 2)^2$$

$$b) y^2 - 25x^2 = (y - 5x)(y + 5x)$$

$$c) (3 + 2x^2)^2 = 9 + 12x^2 + 4x^4$$

$$4.- 2(x-1)^2 + 4x - (x+1)^2 - x^2 = 1 - 2x$$

$$2(x^2 - 2x + 1) + 4x - (x^2 + 2x + 1) - x^2 = 1 - 2x$$

$$2x^2 - 4x + 2 + 4x - x^2 - 2x - 1 - x^2 = 1 - 2x \rightarrow -2x + 1 = 1 - 2x \rightarrow 0x = 0$$

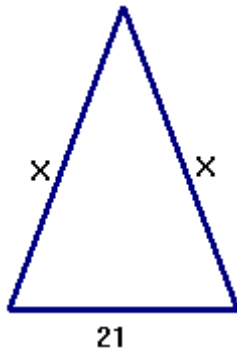
INFINITE SOLUTIONS (IT IS AN IDENTITY)

$$5.- \frac{2x}{3} - \frac{x-2}{5} = 1 - \frac{(x-3)^2}{15} \rightarrow \frac{10x}{15} - \frac{3(x-2)}{15} = \frac{15}{15} - \frac{(x^2 - 6x + 9)}{15}$$

$$10x - 3x + 6 = 15 - x^2 + 6x - 9 \rightarrow x^2 + 10x - 3x - 6x + 6 - 15 + 9 = 0$$

$$x^2 + x = 0 \rightarrow x(x+1) = 0 \rightarrow \begin{cases} x = 0 \\ x + 1 = 0 \rightarrow x = -1 \end{cases}$$

6.- The measure of the unequal side of an isosceles triangle is 21 cm. If the perimeter is 87 cm, how long are the other sides?



$$\text{Perimeter} = x + x + 21 = 87$$

$$2x + 21 = 87$$

$$2x = 66$$

$$x = \frac{66}{2} = 33$$

The other sides are 33 cm long

7.- Bob is five times as old as Carlos and in 8 years' time he will be three times as old as him. How old is each one?

	Now	In 8 years' time
Bob	$5x$	$5x+8$
Carlos	$x$	$x+8$

$$5x + 8 = 3(x + 8) \Rightarrow 5x + 8 = 3x + 24 \Rightarrow 5x - 3x = 24 - 8 \Rightarrow 2x = 16 \Rightarrow x = 8$$

Carlos is 8 years old and Bob is 40 years old

8.- Two cyclists leave at 9 in the morning to from two towns (A and B) located at 150 km away from each other. The cyclist that leaves from A goes at 34 km/h and the one that leaves from B at 26 km/h. How far from A and B will they meet up? When will they meet up?

	Cyclist from A	Cyclist from B
Velocity	34 km/h	26 km/h
Distance	$x$	$150 - x$
time	$t$	$t$

$$t = \frac{x}{34}; \quad t = \frac{150 - x}{26} \Rightarrow \frac{x}{34} = \frac{150 - x}{26} \Rightarrow 26x = 34(150 - x)$$

$$26x = 5100 - 34x \Rightarrow 26x + 34x = 5100 \Rightarrow 60x = 5100 \Rightarrow x = \frac{5100}{60} = 85$$

They meet up at 85 km from A and 65 km from B

$$t = \frac{x}{34} = \frac{85}{34} = 2.5 \Rightarrow 2\text{h}30\text{m}$$

They meet up at 11:30 a.m.

9.- A hotel has double rooms and single rooms. In total there are 58 rooms and 100 beds. How many rooms are there of each type?

Double rooms -  $x$     Single rooms -  $y$

$$\left. \begin{array}{l} x + y = 58 \\ 2x + y = 100 \end{array} \right\} \rightarrow \left. \begin{array}{l} -x - y = -58 \\ 2x + y = 100 \end{array} \right\} \rightarrow x = 42$$

$$42 + y = 58 \rightarrow y = 58 - 42 = 16$$

Solution: There are 42 double rooms and 16 single rooms in the hotel

10.-

$$\left. \begin{array}{l} 3 + 4x = 2y \\ 2x - y = -1 \end{array} \right\} \rightarrow \left. \begin{array}{l} 4x - 2y = -3 \\ 2x - y = -1 \end{array} \right\} \text{Substitution:}$$

$$\left. \begin{array}{l} 2x + 1 = y \\ 4x - 2(2x + 1) = 2 \end{array} \right\} \rightarrow 0x = 4 \text{ No solution}$$

Graphing:

$$\left. \begin{array}{l} y = \frac{4x + 3}{2} \\ y = 2x + 1 \end{array} \right\} \text{parallel lines, inconsistent system}$$

