

## 3<sup>rd</sup> Term General Exam

Remember: in each question write the steps you have taken to reach the solution.  
(1 point each question)

1. Calculate and simplify:

a)  $\frac{2}{3} \cdot \left( \frac{3}{5} - \frac{1}{2} \right) + \frac{7}{5} \cdot \frac{1}{3} =$

b)  $\frac{1}{3} \div \left( 1 - \frac{4}{3} \right)^2 - \left( \frac{1}{3} \right)^{-2} =$

2. Calculate (giving the answer in standard form with 3 s.f.):

a)  $3.078 \cdot 10^{-5} : 1256 =$

b) 15% of 35670041 =

c) 1900 km + 2560 dam =

cm

3. Marta goes to a restaurant and orders a menu. On paying, the waiter tells her that the bill will go up to €26.5, taxes included. If taxes in restaurants are 6%, what is the price of her meal without taxes? Marta decides to leave a 10% tip to the waiter, How much is the tip?

4. Calculate the cost of the hardwood floor to cover a basketball court that measures 26m long and 15m wide if the price of the hardwood floor is €22 per square metre. If I have to pay in advance the 20% of the total price, how much do I have to pay in advance?



5. The third and fifth terms of an arithmetic progression are 470 and 434 respectively. Find the first term and the common difference. Find the sum of the 20 first terms.

6. Solve the equations:

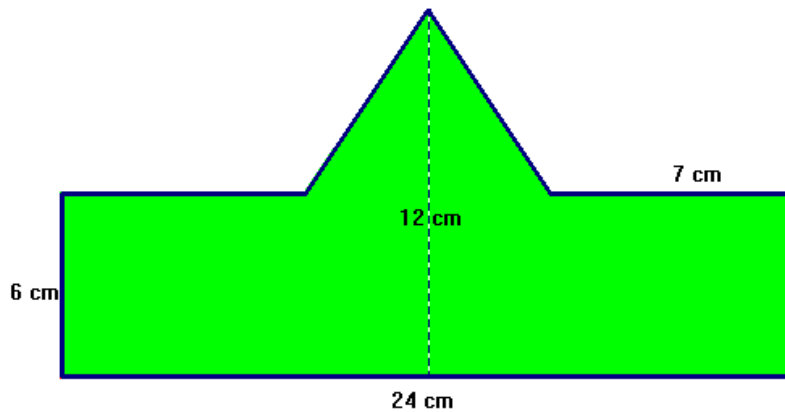
a)  $\frac{(x-1)(x+1)}{2} - \frac{x-5}{6} = \frac{2(x+1)}{3}$

b)  $(2x-1)^2 + 4 = (x+2)^2 + 1$

7. Solve the following system by two different methods: graphing and addition or substitution.

$$\left. \begin{array}{l} 3(2x - y) - 2 = y \\ 4x - 3(y - 1) = 4 \end{array} \right\}$$

8. Find the area and perimeter of the shape:



9. Make a mix of coffee that would cost £6 per kilo with two different types of coffee; one costs £4.50 and the other costs £7. Find the necessary amount of each coffee to make 20 kg of the mix.

10. The table shows data of heights of people in a sample of people in 3<sup>rd</sup> ESO:

Height (cm)	Frequency
[130, 140)	7
[140, 150)	12
[150, 160)	16
[160, 170)	5
[170, 180)	2

- Find the range of heights.
- Find the median class.
- What is the modal class?
- Find the mean height.
- Draw a histogram to represent the data.

**SOLUTION**

1. Calculate and simplify:

$$a) \frac{2}{3} \cdot \left( \frac{3}{5} - \frac{1}{2} \right) + \frac{7}{5} \cdot \frac{1}{3} = \frac{2}{3} \cdot \left( \frac{6}{10} - \frac{5}{10} \right) + \frac{7}{15} = \frac{2}{3} \cdot \frac{1}{10} + \frac{7}{15} = \frac{2}{30} + \frac{7}{15} = \frac{1}{15} + \frac{7}{15} = \frac{8}{15}$$

$$b) \frac{1}{3} \div \left( 1 - \frac{4}{3} \right)^2 - \left( \frac{1}{3} \right)^{-2} = \frac{1}{3} \div \left( -\frac{1}{3} \right)^2 - \left( \frac{3}{1} \right)^2 = \frac{1}{3} \div \frac{1}{9} - 9 = \frac{9}{3} - 9 = -\frac{18}{3} = -6$$

2. Calculate (giving the answer in standard form with 3 s.f.):

$$a) 3.078 \cdot 10^{-5} : 1256 = 2.45 \cdot 10^{-8}$$

$$b) 15\% \text{ of } 35670041 = \frac{15 \cdot 35670041}{100} = 5.35 \cdot 10^6$$

$$c) 1900 \text{ km} + 2560 \text{ dam} = 190000000 \text{ cm} + 2560000 \text{ cm} = 1.93 \cdot 10^8 \text{ cm}$$

3. Marta goes to a restaurant and orders a menu. On paying, the waiter tells her that the bill will go up to €26.5, taxes included. If taxes in restaurants are 6%, what is the price of her meal without taxes? Marta decides to leave a 10% tip to the waiter, How much is the tip?

$$\frac{26.5}{106} = \frac{x}{100} \Rightarrow 106x = 2650 \Rightarrow x = 25$$

The price without taxes is €25.

$$\frac{26.5}{100} = \frac{x}{10} \Rightarrow x = 2.65 \text{ The tip is €2.65}$$

4. Calculate the cost of the hardwood floor to cover a basketball court that measures 26m long and 15m wide if the price of the hardwood floor is €22 per square metre. If I have to pay in advance the 20% of the total price, how much do I have to pay in advance?



$$A = b \cdot a = 26 \times 15 = 390 \text{ m}^2$$

$$390 \times 22 = 8580 \text{ The cost of the hardwood floor is €8580}$$

$$20\% \text{ of } 8580 \rightarrow \frac{8580 \times 20}{100} = 1716$$

I have to pay in advance €1716

5. The third and fifth terms of an arithmetic progression are 470 and 434 respectively. Find the first term and the common difference. Find the sum of the 20 first terms.

$$a_5 = a_3 + 2d \rightarrow 434 = 470 + 2d \rightarrow 2d = -36 \rightarrow d = -18$$

$$a_3 = a_1 + 2d \rightarrow 470 = a_1 + 2 \times (-18) \rightarrow a_1 = 470 + 36 \rightarrow a_1 = 506$$

$$a_{20} = a_1 + 19d \rightarrow a_{20} = 506 + 19 \times (-18) \rightarrow a_{20} = 164$$

$$S_{20} = \frac{(a_1 + a_{20}) \cdot 20}{2} = \frac{(506 + 164)20}{2} = 6700$$

6. Solve the equations:

$$a) \frac{(x-1)(x+1)}{2} - \frac{x-5}{6} = \frac{2(x+1)}{3} \rightarrow \frac{x^2-1}{2} - \frac{x-5}{6} = \frac{2x+2}{3}$$

$$\frac{3(x^2-1)}{6} - \frac{x-5}{6} = \frac{2(2x+2)}{6} \rightarrow 3x^2 - 3 - x + 5 = 4x + 4 \rightarrow 3x^2 - 5x - 2 = 0$$

$$x = \frac{5 \pm \sqrt{25 + 24}}{6} = \frac{5 \pm 7}{6} = \begin{cases} 6 \\ -\frac{1}{3} \end{cases}$$

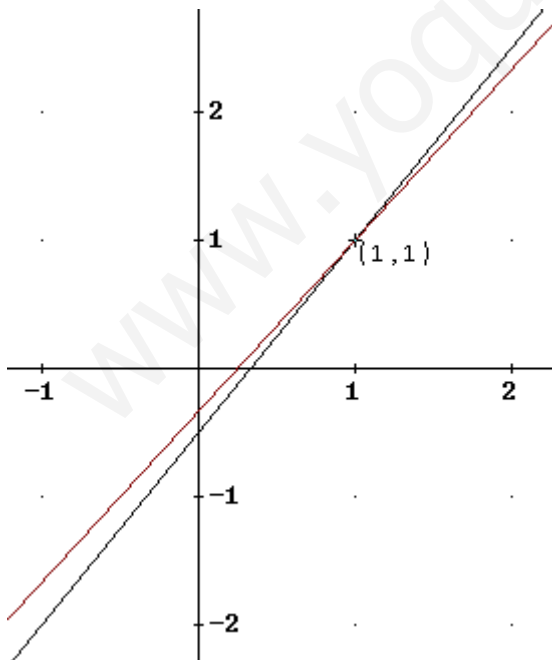
$$b) (2x-1)^2 + 4 = (x+2)^2 + 1 \rightarrow 4x^2 - 4x + 1 + 4 = x^2 + 4x + 4 + 1$$

$$3x^2 - 8x = 0 \rightarrow x(3x-8) = 0 \rightarrow \begin{cases} x = 0 \\ 3x-8 = 0 \rightarrow x = \frac{8}{3} \end{cases}$$

7. Solve the following system by two different methods: graphing and addition or substitution.

$$\left. \begin{array}{l} 3(2x-y) - 2 = y \\ 4x - 3(y-1) = 4 \end{array} \right\} \rightarrow \left. \begin{array}{l} 6x - 3y - 2 = y \\ 4x - 3y + 3 = 4 \end{array} \right\} \rightarrow \left. \begin{array}{l} 6x - 2 = 4y \\ 4x - 1 = 3y \end{array} \right\} \rightarrow \left. \begin{array}{l} y = \frac{6x-2}{4} \\ y = \frac{4x-1}{3} \end{array} \right\}$$

Graphing:



Substitution:

$$\left. \begin{array}{l} 6x - 3y - 2 = y \\ 4x - 3y + 3 = 4 \end{array} \right\} \rightarrow \left. \begin{array}{l} y = \frac{6x-2}{4} \\ 4x - 3y = 1 \end{array} \right\}$$

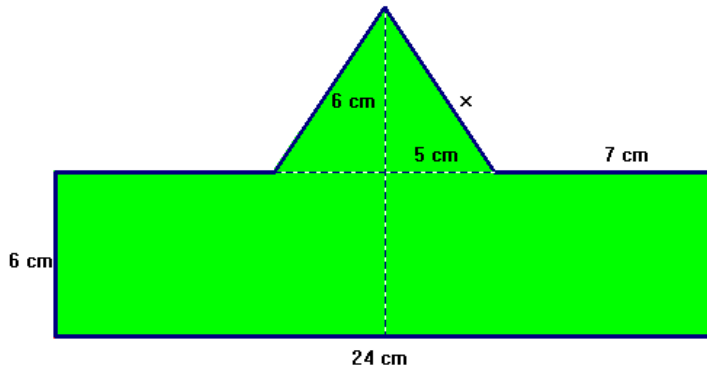
$$4x - 3 \cdot \frac{6x-2}{4} = 1 \rightarrow 4x - \frac{9x-3}{2} = 1$$

$$8x - 9x + 3 = 2 \rightarrow -x = 2 - 3 \rightarrow x = 1$$

$$y = \frac{6x-2}{4} = \frac{6-2}{4} = 1$$

Solution:  $x = 1, y = 1$

8. Find the area and perimeter of the shape:



Perimeter:

$$x^2 = 6^2 + 5^2 = 61$$

$$x = \sqrt{61} = 7.81$$

$$P = 24 + 6 + 6 + 7 + 7 + 7.81 + 7.81$$

$$P = 65.62 \text{ cm}$$

Area: Triangle  $\rightarrow A = \frac{b \cdot a}{2} = \frac{10 \cdot 6}{2} = 30 \text{ cm}^2$ ; Rectangle:  $A = 24 \cdot 6 = 144 \text{ cm}^2$

$$\text{Area} = 30 + 144 = 154 \text{ cm}^2$$

9. Make a mix of coffee that would cost £6 per kilo with two different types of coffee; one costs £4.50 and the other costs £7. Find the necessary amount of each coffee to make 20 kg of the mix.

	Quantity	Prize	Cost
Coffee 1	x	4.50 €/k	4.50x
Coffee 2	20-x	7 €/k	7(20-x)
Mix	20 k	6 €/k	120

$$4.50x + 7(20 - x) = 120 \Rightarrow 4.50x + 140 - 7x = 120 \Rightarrow -2.50x = -20 \Rightarrow x = 8$$

We need 8 k of coffee 1 (4.50€/k) and 12 k of coffee 2 (7 €/k)

10. The table shows data of heights of people in a sample of people in 3<sup>rd</sup> ESO:

Height (cm)	$x_i$	$f_i$	$F_i$	$x_i \cdot f_i$
[130, 140)	135	7	7	945
[140, 150)	145	12	19	1740
[150, 160)	155	16	35	2480
[160, 170)	165	5	40	825
[170, 180)	175	2	42	350

a) Find the range of heights.  $180 - 130 = 50 \text{ cm}$

b) Find the median class. [150,160)

c) What is the modal class? [150,160)

d) Find the mean height  $\bar{x} = \frac{\sum x_i \cdot f_i}{N} = \frac{6340}{42} = 150.95 \text{ cm}$

e) Draw a histogram to represent the data.

