

EXAM UNIT 5 (EQUATIONS)

1) Solve the following equations: (4 points)

a. $2x - 4(x - 1) + 2x - 1 = 2(x + 5) - 2x$

b. $\frac{2x - 5}{5} = 2 - x - \frac{x + 1}{4}$

c. $(3x - 1)^2 + 6x - 8 = (3x - 2)(3x + 2) - 3$

d. $\frac{x - 10}{2} - \frac{x - 20}{4} + \frac{x - 30}{3} = 5$

2) Solve the following equations: (5 points)

a. $(x - 2)^2 - 3 = 3x + 1$

b. $x(x - 3) = 2 - 3x - (x + 4)(x - 4)$

c. $2x + 3(x - 4)^2 = 37 + (x - 3)(x + 3)$

d. $\frac{(x - 2)(x + 2)}{4} - \frac{x - 6}{2} = \frac{10 + x}{5}$

e. $\frac{(x - 1)^2 - 3x + 1}{15} + \frac{x + 1}{5} = 0$

3) Translate into an equation: (1 point)

a. Three subtracted from twice a number is equal to seventeen.

b. One third of a number plus five is equal to ten.

c. A number plus four, all multiplied by seven is equal to forty-two.

SOLUTION

1) Solve the following equations:

a. $2x - 4(x - 1) + 2x - 1 = 2(x + 5) - 2x \rightarrow 2x - 4x + 4 + 2x - 1 = 2x + 10 - 2x$
 $2x - 4x + 2x - 2x + 2x = 10 - 4 + 1 \rightarrow 0x = 7$ It hasn't got any solution

b. $\frac{2x-5}{5} = 2 - x - \frac{x+1}{4} \rightarrow \frac{8x-20}{20} = \frac{40}{20} - \frac{20x}{20} - \frac{5x+5}{20}$
 $8x - 20 = 40 - 20x - 5x - 5 \rightarrow 8x + 20x + 5x = 40 - 5 + 20$
 $33x = 55 \rightarrow x = \frac{55}{33} \rightarrow x = \frac{5}{3}$

c. $(3x-1)^2 + 6x - 8 = (3x-2)(3x+2) - 3 \rightarrow 9x^2 - 6x + 1 + 6x - 8 = 9x^2 - 4 - 3$
 $9x^2 - 9x^2 - 6x + 1 + 6x - 8 = -4 - 3 \rightarrow 0x = -4 - 3 + 8 - 2 \rightarrow 0x = 0$
 It is an identity (Solution: all real numbers)

d. $\frac{x-10}{2} - \frac{x-20}{4} + \frac{x-30}{3} = 5 \rightarrow \frac{6x-60}{12} - \frac{3x-60}{12} + \frac{4x-120}{12} = \frac{60}{12}$
 $6x - 60 - 3x + 60 + 4x - 120 = 60 \rightarrow 7x = 60 + 120 \rightarrow x = \frac{180}{7}$

2) Solve the following equations:

a. $(x-2)^2 - 3 = 3x + 1 \rightarrow x^2 - 4x + 4 - 3 = 3x + 1 \rightarrow x^2 - 7x = 0$
 $x^2 - 7x = 0 \rightarrow x(x-7) = 0 \begin{cases} x = 0 \\ x - 7 = 0 \end{cases} \rightarrow \begin{cases} x_1 = 0 \\ x_2 = 7 \end{cases}$

b. $x(x-3) = 2 - 3x - (x+4)(x-4) \rightarrow x^2 - 3x = 2 - 3x - (x^2 - 16)$
 $x^2 - 3x = 2 - 3x - x^2 + 16 \rightarrow 2x^2 = 18 \rightarrow x^2 = 9 \rightarrow x = \pm\sqrt{9} = \pm 3$

c. $2x + 3(x-4)^2 = 37 + (x-3)(x+3) \rightarrow 2x + 3(x^2 - 8x + 16) = 37 + x^2 - 9$
 $2x + 3x^2 - 24x + 48 = 37 + x^2 - 9 \rightarrow 2x^2 - 22x + 20 = 0 \rightarrow x^2 - 11x + 10 = 0$
 $x = \frac{11 \pm \sqrt{11^2 - 40}}{2} = \frac{11 \pm \sqrt{81}}{2} = \frac{11 \pm 9}{2} \begin{cases} x_1 = 10 \\ x_2 = 1 \end{cases}$

d. $\frac{(x-2)(x+2)}{4} - \frac{x-6}{2} = \frac{10+x}{5} \rightarrow \frac{x^2-4}{4} - \frac{x-6}{2} = \frac{10+x}{5}$
 $\frac{5x^2-20}{20} - \frac{10x-60}{20} = \frac{40+4x}{20} \rightarrow 5x^2 - 20 - 10x + 60 = 40 + 4x$
 $5x^2 - 14x = 0 \rightarrow x(5x-14) = 0 \rightarrow \begin{cases} x_1 = 0 \\ 5x-14 = 0 \rightarrow x_2 = \frac{14}{5} \end{cases}$

$$e. \frac{(x-1)^2 - 3x + 1}{15} + \frac{x+1}{5} = 0 \rightarrow \frac{x^2 - 2x + 1 - 3x + 1}{15} + \frac{x+1}{5} = 0$$

$$\frac{x^2 - 5x + 2}{15} + \frac{3x + 3}{15} = 0 \rightarrow x^2 - 5x + 2 + 3x + 3 = 0 \rightarrow x^2 - 2x + 5 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 20}}{2} = \frac{2 \pm \sqrt{-16}}{2} \text{ It does not have any solution}$$

3) Translate into an equation:

a. Three subtracted from twice a number is equal to seventeen.

$$2x - 3 = 17$$

b. One third of a number plus five is equal to ten.

$$\frac{x}{3} + 5 = 10$$

c. A number increased by four, all multiplied by seven equals forty-two.

$$7(x + 4) = 42$$