6. **a) Siendo** $z = 1 - 3i$ **y** $w = \sqrt{8}_{45^\circ}$**, calcular:** $z + w, z \cdot w, 2z, \dfrac{z^5}{w}$

   b) Calcula: $\dfrac{i^{35} - i^5}{2i^{10}}$

7. a) Opera: $\dfrac{(1-2i)(-2+i)}{3i(1-i)}$

   b) Resuelve la siguiente ecuación y expresa el resultado en forma binómica:

   $\dfrac{3 - zi + 2i}{2} = z + i$

8. Siendo $z = 4\sqrt{3} - 4i$**, se pide:**

   a) z^5

   b) Las raíces cúbicas de z

9. Halla el valor de x para que el cociente (1 + 3xi) : (3 –4i)

   a) Sea un número imaginario puro.

   b) Sea un número real.

   c) Tenga módulo 1

10. Encontrar a y b para que $\dfrac{a - 6i}{3 + bi} = \sqrt{2}_{315}$

1. Calcular: $\sqrt{-2+2\sqrt{3}i} =$

2. Calcular: $a)\left(1-\dfrac{\sqrt{3}-i}{2}\right)^{12}$ $b)\left(\dfrac{1+\sqrt{3}i}{1-i}\right)^{\frac{1}{4}}$

3. Expresar en forma binómica el resultado de: $a)\left(\dfrac{3\sqrt{3}}{2}+\dfrac{3}{2}i\right)^{5}$ $b)(-1+i)^{10}$

4. Calcular: $\sqrt[5]{\dfrac{-8-8\sqrt{3}i}{\left(-2\sqrt{3}\right)^{2}}}+2i$

5. Resolver:

$$\left.\begin{array}{l} z^4 + 27z = 0 \\ z^5 + 125\,z^2 = 0 \end{array}\right\}$$

6. Resolver, expresando el resultado de forma binómica y trigonométrica: $\sqrt[3]{\dfrac{5+i}{2+3i}}$

7. Resolver: $a)\sqrt[3]{\dfrac{1-i}{1+i}}$ $b)\sqrt[3]{\dfrac{-1+i}{1+\sqrt{3}i}}$ $c)\sqrt[4]{81(\cos 120^{o}+isen120^{o})}$

8. Siendo $z=1-3i$ y $w=\sqrt{8}_{45^{o}}$, calcular: *z + w, z·w, 2z,* $\dfrac{z^3}{w}$

9. Calcula: $\dfrac{i^{35}-i^{5}}{2\cdot i^{10}}$

10. Opera: $\dfrac{(1-2i)(-2+i)}{3i\cdot(1-i)}$

11. Resuelve la siguiente ecuación y expresa el resultado en forma binómica: $\dfrac{3-zi+2i}{2}=z+i$

12. Siendo $z=4\sqrt{3}-4i$, se pide:

a) z^5

b) Las raíces cúbicas de z.

13. Hallar el valor de x para que el cociente $\dfrac{1+3xi}{3-4i}$:

a) Sea un número imaginario puro.
b) Sea un número real.
c) Tenga módulo 1.

14. Encontrar a y b para que $\dfrac{a-6i}{3+bi}=\sqrt{2}_{315^{o}}$

NÚMEROS COMPLEJOS

6 a) $z = 1-3i$, $W = \sqrt{8}_{45^\circ} = 2+2i$

$\downarrow$

$z = \sqrt{10}_{288'4^\circ}$

$z+w = (1-3i)+(2+2i) = 3-i$

$z \cdot w = (1-3i)\cdot(2+2i) = 8-4i$

$2z = 2-6i$

$$\frac{z^3}{w} = \frac{(1-3i)^3}{2+2i} = \frac{-26+18i}{2+2i} = -2+11i$$

b) $$\frac{i^{35}-i^5}{2\cdot i^{10}} = \frac{(-i)-i}{2\cdot(-1)} = \frac{-2i}{-2} = i$$

$35 \,\underline{|4}$; $3 \;\; 8$ — $5 \,\underline{|4}$; $1 \;\; 1$ — $10 \,\underline{|4}$; $2 \;\; 2$

	R
$i^1 = i$	1
$i^2 = -1$	2
$i^3 = -i$	3
$i^4 = 1$	0

---

7 a) $$\frac{(1-2i)(-2+i)}{3i(1-i)} = \frac{5i}{3+3i} = \frac{5}{6}+\frac{5}{6}i$$

b) $$\frac{3-zi+2i}{2} = z+i \rightarrow 3-zi+2i = 2z+2i \rightarrow 3 = 2z+zi$$

$$3 = (2+i)z \rightarrow z = \frac{3}{2+i}\cdot\frac{2-i}{2-i} = \frac{6-3i}{5} = \frac{6}{5}-\frac{3}{5}i$$

---

8 $z = 4\sqrt{3}-4i = 8_{330^\circ}$

a) $z^5 = (8_{330})^5 = 8^5{}_{330\cdot 5} = 32768_{1650^\circ} = 32768_{210^\circ}$

b) $$\sqrt[3]{z} = \sqrt[3]{8_{330^\circ}} = \sqrt[3]{8}_{\frac{330+k360}{3}} = 2\begin{cases}110^\circ\\230^\circ\\350^\circ\end{cases}$$

---

(9) $\frac{1+3xi}{3-4i}\cdot\frac{3+4i}{3+4i}=\frac{(3-12x)+(9x+4)i}{25}$

a) $\frac{3-12x}{25}=0 \quad x=\frac{1}{4}$

b) $\frac{9x+4}{25}=0 \quad x=\frac{-4}{9}$

c) $\sqrt{\left(\frac{3-12x}{25}\right)^2+\left(\frac{9x+4}{25}\right)^2}=1 \rightarrow x=\pm\sqrt{\frac{8}{3}}$

---

(10) $\frac{a-6i}{3+bi}=\sqrt{2}_{315}$

$\frac{a-6i}{3+bi}\cdot\frac{3-bi}{3-bi}=\frac{3a-18i-abi+6bi^2}{9+b^2}=\frac{(3a-6b)}{9+b^2}+\frac{(-18-ab)i}{9+b^2}$

$\sqrt{2}_{315^\circ}=\sqrt{2}\cos 315+\sqrt{2}\,\text{sen}\,315i=\sqrt{2}\cdot\frac{\sqrt{2}}{2}-\sqrt{2}\cdot\frac{\sqrt{2}}{2}i=1-i$

$\Rightarrow \left.\begin{array}{l}\frac{3a-6b}{9+b^2}=1\\ \frac{-18-ab}{9+b^2}=-1\end{array}\right\}\left.\begin{array}{l}3a-6b=9+b^2\\ -18-ab=-9-b^2\end{array}\right\}\begin{array}{l}a=\frac{9+b^2+6b}{3}\\ -18-\left(\frac{9+b^2+6b}{3}\right)b=-9-b^2\end{array}$

$54+9b+b^3+6b^2=-27-3b^2 \rightarrow b^3+9b^2+9b+81=0$

$$\begin{array}{r|rrrr} & 1 & 9 & 9 & 81\\ -9 & & -9 & 0 & -81\\ \hline & 1 & 0 & 9 & \lfloor 0\end{array}$$

$\boxed{\begin{array}{l}b=-9\rightarrow a=12\\ b=3i\rightarrow a=6i\\ b=-3i\rightarrow a=-6i\end{array}}$

$x^2+9=0$

$x^2=-9$

$x=\pm\sqrt{-9}=\pm 3i$

---

① $\sqrt{-2+2\sqrt{3}i} = \sqrt{4_{120^\circ}} = \sqrt{4}\left\{\begin{array}{l}\frac{120+k360}{2} \\ k=0,1\end{array}\right. =$

$= 2_{60}\ ,\ 2_{240^\circ}$

② a) $\left(1-\frac{\sqrt{3}-i}{2}\right)^{12} = \left(\frac{2-\sqrt{3}+i}{2}\right)^{12} = \left(\frac{2-\sqrt{3}}{2}+\frac{1}{2}i\right)^{12} =$

$= \left(\sqrt{2-\sqrt{3}}_{\,75^\circ}\right)^{12} = \left(\sqrt{2-\sqrt{3}}\right)^{12}_{\,75\cdot 12} = (2-\sqrt{3})^6_{900^\circ} =$

$= (2-\sqrt{3})^6_{180^\circ}$

b) $\left(\frac{1+\sqrt{3}i}{1-i}\right)^{1/4} = \sqrt[4]{\frac{2_{60^\circ}}{\sqrt{2}_{315}}} = \sqrt[4]{\frac{2}{\sqrt{2}}_{-255^\circ}} = \sqrt[4]{\sqrt{2}_{105^\circ}} =$

$= \sqrt[8]{2}\left\{\begin{array}{l}\frac{105+k360}{4} \\ k=0,1,2,3\end{array}\right.$ $\quad \sqrt[8]{2}_{26'25^\circ}, \sqrt[8]{2}_{116'25^\circ}$

$\sqrt[8]{2}_{206'25^\circ}, \sqrt[8]{2}_{296'25^\circ}$

③ a) $\left(\frac{3\sqrt{3}}{2}+\frac{3}{2}i\right)^5=\left(3_{30^\circ}\right)^5=3^5{}_{150^\circ}=$

$243_{150^\circ}=243(\cos 150^\circ+i\,\text{sen}\,150^\circ)=$

$=243\left(\frac{-\sqrt{3}}{2}+\frac{1}{2}i\right)=\frac{-243\sqrt{3}}{2}+\frac{243}{2}i$

b) $(-1+i)^{10}=\left(\sqrt{2}_{135^\circ}\right)^{10}=\left(\sqrt{2}\right)^{10}{}_{1350^\circ}=$

$=2^5{}_{270}=32_{270}=32(\cos 270+\text{sen}\,270\,i)=0-32i$

$=-32i$

---

④ $x=\sqrt[5]{\frac{-8-8\sqrt{3}i}{(-2\sqrt{3})^2}}+2i$

Calculemos a parte la raíz:

$y=\sqrt[5]{\frac{16_{240^\circ}}{12}}=\sqrt[5]{\frac{4}{3}_{240^\circ}}=\sqrt[5]{\frac{4}{3}}\begin{cases}\frac{240+k360}{5}\\ k=0,1,2,3,4\end{cases}$

$x_1=y_1+2i=\sqrt[5]{\frac{4}{3}}_{48^\circ}+2i=\sqrt[5]{\frac{4}{3}}(\cos 48+i\,\text{sen}\,48)+2i$

$x_2=\sqrt[5]{\frac{4}{3}}_{120^\circ}+2i=\ldots$

$x_3=\sqrt[5]{\frac{4}{3}}_{192^\circ}+2i=\ldots$

$x_4=\sqrt[5]{\frac{4}{3}}_{264^\circ}+2i=\ldots$

$x_5=\sqrt[5]{\frac{4}{3}}_{336^\circ}+2i=\ldots$

Calculadora y decimales

¡¡¿Quién se inventa estos problemas para un examen?!!

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⑤ $\begin{cases} z^4+27z=0 \\ z^5+125z^2=0 \end{cases}$

• $z^4+27z=0 \quad z(z^3+27)=0 \quad \begin{matrix} z=0 \\ z^3+27=0 \end{matrix}$

$$z=\sqrt[3]{-27}=\sqrt[3]{27_{180°}}=\sqrt[3]{27}\begin{cases}\frac{180+k360}{3} \\ k=0,1,2\end{cases}=$$

$$=\begin{cases}3_{60°}\\3_{180°}\\3_{300°}\end{cases}$$

• $z^5+125z^2=0 \quad z^2=0$

$z^2(z^3+125)=0 \quad z^3=-125 \quad z=\sqrt[3]{-125}=\sqrt[3]{125_{180°}}$

$$=5\begin{cases}\frac{180+k360}{3}\\k=0,1,2\end{cases}=\begin{cases}5_{60°}\\5_{180°}\\5_{300°}\end{cases}$$

Solución del sistema: $z=0$

———o———

⑥ • $\sqrt[3]{\frac{5+i}{2+3i}}=\sqrt[3]{\sqrt{26}}=\sqrt[3]{\sqrt{2}_{315°}}=\sqrt[6]{2}\begin{cases}\frac{315°+k360°}{3}\\k=0,1,2\end{cases}=$

$=\sqrt[6]{2}_{105°}, \sqrt[6]{2}_{225°}, \sqrt[6]{2}_{345°}$

———o———

7) a) $\sqrt[3]{\frac{1-i}{1+i}} = \sqrt[3]{-i} = \sqrt[3]{1_{270°}} = \sqrt[3]{1} \left\{ \begin{array}{l} \frac{270+k360}{3} \\ k=0,1,2 \end{array} \right. =$

$= 1_{90°}, 1_{210°}, 1_{330°}$

b) $\sqrt[3]{\frac{-1+i}{1+\sqrt{3}i}} = \sqrt[3]{\frac{\sqrt{2}_{135°}}{2_{60°}}} = \sqrt[3]{\frac{\sqrt{2}}{2}_{75°}} = \sqrt[3]{\frac{\sqrt{2}}{2}} \left\{ \begin{array}{l} \frac{75+k360}{3} \\ k=0,1,2 \end{array} \right. =$

$= \sqrt[3]{\frac{\sqrt{2}}{2}} \left\{ \begin{array}{l} 25° \\ 145° \\ 265° \end{array} \right.$

c) $\sqrt[4]{81(\cos 120° + i \operatorname{sen} 120°)} = \sqrt[4]{81_{120°}} = \sqrt[4]{81} \left\{ \begin{array}{l} \frac{120+k360}{4} \\ k=0,1,2,3 \end{array} \right. =$

$= 3_{30°}, 3_{120°}, 3_{210°}, 3_{300°}$

---

8) $z = 1-3i \quad , w = \sqrt{8}_{45°} = \sqrt{8}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 2+2i$

$z+w = (1-3i)+(2+2i) = 3-i$

$z \cdot w = 8-4i$

$2 \cdot z = 2-6i$

$\frac{z^3}{w} = \frac{(1-3i)^3}{2+2i} = \frac{-26+18i}{2+2i} = -2+11i$

---

9) $\frac{i^{35}-i^5}{2 \cdot i^{10}} = \frac{(-i)-i}{2(-1)} = \frac{-2i}{-2} = i$

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10) $\frac{(1-2i)(-2+i)}{3i(1-i)} = \frac{5i}{3+3i} = \frac{5}{6} + \frac{5}{6}i$

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(11) $\frac{3-zi+2i}{2} = z+i \rightarrow 3-zi+2i = 2z+2i \rightarrow$

$2z+zi = 3+2i-2i \rightarrow (2+i)z = 3 \rightarrow z = \frac{3}{2+i} \rightarrow$

$z = \frac{3(2-i)}{(2+i)(2-i)} = \frac{6-3i}{5} = \frac{6}{5} - \frac{3}{5}i$

(12) $z = 4\sqrt{3} - 4i = 8_{330°}$

a) $z^5 = (8_{330°})^5 = 8^5_{330 \cdot 5} = 32768_{1650°} =$

$= 32768_{210°}$

b) $\sqrt[3]{4\sqrt{3}-4i} = \sqrt[3]{8_{330°}} = \sqrt[3]{8} \begin{cases} \frac{330°+k360}{2} \\ k=0,1,2 \end{cases} =$

$= 2_{165°},\ 2_{285°},\ 2_{405°}$

(13) $\frac{1+3xi}{3-4i} \cdot \frac{(3+4i)}{(3+4i)} = \frac{3+9xi+4i+12xi^2}{9+16} = \frac{(3-12x)}{25} + \frac{(9x+4)i}{25}$

a) $\frac{3-12x}{25} = 0 \rightarrow x = \frac{1}{4}$; b) $\frac{9x+4}{25} = 0 \quad x = \frac{-4}{9}$

c) $\sqrt{\left(\frac{3-12x}{25}\right)^2 + \left(\frac{9x+4}{25}\right)^2} = 1 \rightarrow x = \pm\sqrt{\frac{8}{3}}$

(14) Igual que el 10 de la otra hoja.