1. Dada la función:

$$y = F(x) = \begin{cases} x^2 - 5 & \text{si} \quad x < 1 \\ 3x + 2 & \text{si} \quad 1 \le x < 4 \\ 4x - 2 & \text{si} \quad x \ge 4 \end{cases}$$

¿Existe el límite de F(x) para $x \to 1$? ¿Y para $x \to 4$? Razona tu respuesta.
Dibuja la gráfica de F(x).

2. Calcular b para que

$$\lim_{x\to+\infty} \frac{-x^2 + 2x}{bx(1 + x)} = \frac{1}{5}$$

3. Calcular los siguientes límites:

a) $\lim\limits_{x\to+\infty} \dfrac{2x^2 - 5x + 6}{x^3 - x^2 + 8}$

b) $\lim\limits_{x\to+\infty} \dfrac{4x^4 - 2x}{x(x^2 - 1)}$

c) $\lim\limits_{x\to+\infty} \left(\dfrac{2x^2 + 1}{x^2 + 5}\right)^{\frac{3x+1}{x+2}}$

d) $\lim\limits_{x\to+\infty} \left(\sqrt{x^3 + 2} - 5x\right)$

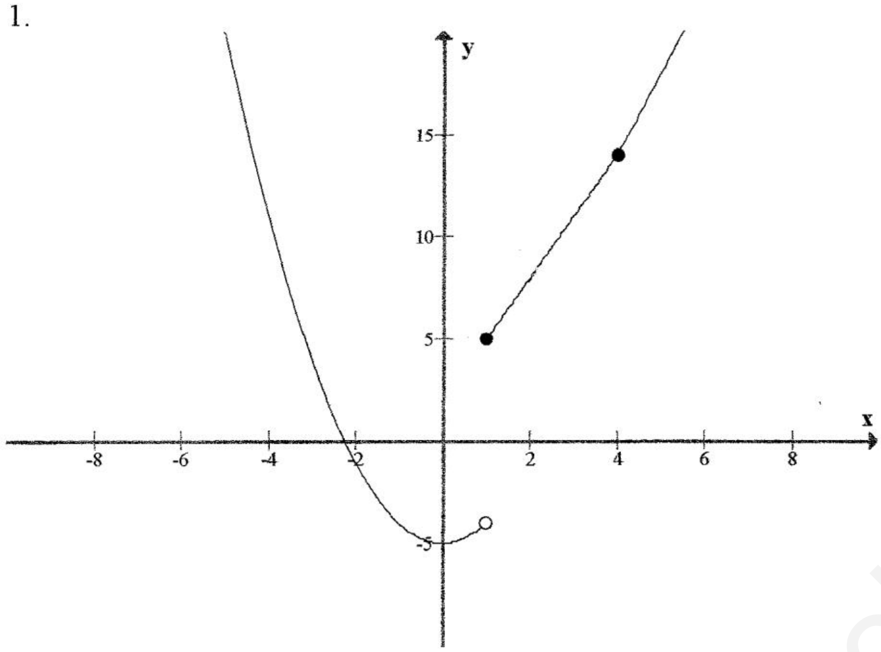
4. Calcular los siguientes límites del número e:

a) $\lim\limits_{x\to 2} \left(\dfrac{x + 3}{2x + 1}\right)^{2/(x-2)}$

b) $\lim\limits_{x\to+\infty} \left(\dfrac{3x + 1}{3x + 4}\right)^{2x}$

SOLUCIONES

1.



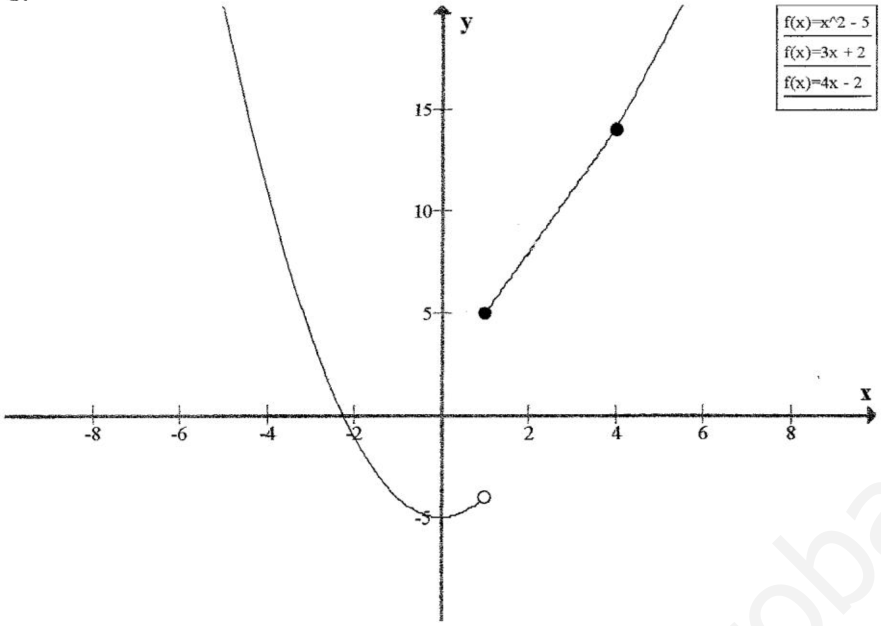
No existe límite para x = 1, discontinuidad de salto finito. Sí existe límite para x = 4, vale 14, es continua.

2. $b = -5$

3. a) 0 b) $\pm\infty$ c) 8 d) $\pm\infty$

4. a) $e^{-2/5}$ b) e^{-2}

1\. 1.



$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2-5 = -4$$

$\neq$ $\nexists$ límite en $x=1$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 3x+2 = 5$$

discontinua de salto finito

$$\lim_{x\to 4^-} 3x+2 = 14 = \lim_{x\to 4^+} 4x-2 = f(4)$$

$\exists$ límite y es continua en $x=4$

——o——

2. $\lim_{x\to\infty} \frac{-x^2+2x}{bx(1+x)} = \frac{1}{5}$

$$\lim_{x\to\infty} \frac{-x^2+2x}{bx(1+x)} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{-x^2+2x}{bx^2+bx} =$$

$$\lim_{x\to\infty} \frac{-\frac{x^2}{x^2}+\frac{2x}{x^2}}{\frac{bx^2}{x^2}+\frac{bx}{x^2}} = \lim_{x\to\infty} \frac{-1+\cancel{\frac{2}{x}}^{\to 0}}{b+\cancel{\frac{b}{x}}_{\to 0}} = \frac{-1}{b} = \frac{1}{5}$$

$$\Downarrow$$

$$\boxed{b=-5}$$

—— o ——

3. a) $\lim_{x\to\infty} \frac{2x^2-5x+6}{x^3-x^2+8} = \left[\frac{\infty}{\infty}\right] =$

$$= \lim_{x\to\infty} \frac{\frac{2x^2}{x^3}-\frac{5x}{x^3}+\frac{6}{x^3}}{\frac{x^3}{x^3}-\frac{x^2}{x^3}+\frac{8}{x^3}} = \lim_{x\to\infty} \frac{\cancel{\frac{2}{x}}^{\to 0}-\cancel{\frac{5}{x^2}}^{\to 0}+\cancel{\frac{6}{x^3}}^{\to 0}}{1-\cancel{\frac{1}{x}}_{\to 0}+\cancel{\frac{8}{x^3}}_{\to 0}} =$$

$$= \frac{0}{1} = 0$$

b) $\lim_{x\to\infty} \frac{4x^4-2x}{x(x^2-1)} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{4x^4-2x}{x^3-x} =$

$$= \lim_{x\to\infty} \frac{\frac{4x^4}{x^4}-\frac{2x}{x^4}}{\frac{x^3}{x^4}-\frac{x}{x^4}} = \lim_{x\to\infty} \frac{4-\cancel{\frac{2}{x^3}}^{\to 0}}{\cancel{\frac{1}{x}}_{\to 0}-\cancel{\frac{1}{x^3}}_{\to 0}} = \frac{4}{0} = \infty$$

3. c) $\lim_{x\to\infty} \left(\frac{2x^2+1}{x^2+5}\right)^{\frac{3x+1}{x+2}} = \left(\frac{\infty}{\infty}\right)^{\left(\frac{\infty}{\infty}\right)} =$

$= \lim_{x\to\infty} \left(\frac{2x^2+1}{x^2+5}\right)^{\frac{3x+1}{x+2}} = 2^3 = 8$

d) $\lim_{x\to\infty} \left(\sqrt{x^3+2x} - 5x\right) = [\infty - \infty] =$

$= \lim_{x\to\infty} \left(\sqrt{x^3+2x} - 5x\right) \frac{\left(\sqrt{x^3+2x}+5x\right)}{\left(\sqrt{x^3+2x}+5x\right)} =$

$= \lim_{x\to\infty} \frac{\left(\sqrt{x^3+2x}\right)^2 - (5x)^2}{\sqrt{x^3+2x}+5x} = \lim_{x\to\infty} \frac{x^3+2x-25x^2}{\sqrt{x^3+2x}+5x} =$

$= \lim_{x\to\infty} \frac{\overbrace{x^3+2x-25x^2}^{\text{grado } 3}}{\underbrace{\sqrt{x^3+2x}+5x}_{\text{grado } \frac{3}{2}}} = \left[\frac{\infty}{\infty}\right] = \infty$

—— o ——

4. a) $\lim_{x \to 2} \left(\frac{x+3}{2x+1} \right)^{\frac{2}{x-2}} = [1^{\infty}] =$

$= e^{\lim_{x \to 2} \frac{2}{x-2} \cdot \left(\frac{x+3}{2x+1} - 1 \right)} =$

$= e^{\lim_{x \to 2} \frac{2}{x-2} \frac{x+3-2x-1}{2x+1}} = e^{\lim_{x \to 2} \frac{2}{x-2} \cdot \frac{-x+2}{2x+1}} =$

$= e^{\lim_{x \to 2} \frac{2}{\cancel{x-2}} \frac{-\cancel{(x-2)}}{2x+1}} = e^{\lim_{x \to 2} \frac{-2}{2x+1}} = e^{-2/5}$

b) $\lim_{x \to \infty} \left(\frac{3x+1}{3x+4} \right)^{2x} = [1^{\infty}] =$

$= e^{\lim_{x \to \infty} 2x \left(\frac{3x+1}{3x+4} - 1 \right)} =$

$= e^{\lim_{x \to \infty} 2x \left(\frac{3x+1-3x-4}{3x+4} \right)} = e^{\lim_{x \to \infty} 2x \left(\frac{-3}{3x+4} \right)} =$

$= e^{\lim_{x \to \infty} \frac{-6x}{3x+4}} = e^{-2}$