

$$\int x e^x dx$$

$$\text{Sol: } e^x(x-1) + k$$

Por el método de integración por partes:

$$\int x e^x dx = \left\{ \begin{array}{l} u = x \Rightarrow du = dx \\ dv = e^x dx \Rightarrow v = e^x \end{array} \right\} = x e^x - \int e^x dx = x e^x - e^x + k = (x-1)e^x + k$$

$$I = \int (x^2 - 3x + 5)e^x dx$$

$$\text{Sol: } e^x(x^2 - 5x + 10) + k$$

Por el método de integración por partes:

$$\begin{aligned} I &= \int (x^2 - 3x + 5)e^x dx = \left\{ \begin{array}{l} u = x^2 - 3x + 5 \Rightarrow du = (2x - 3)dx \\ dv = e^x dx \Rightarrow v = e^x \end{array} \right\} = \\ &= (x^2 - 3x + 5) e^x - \int e^x(2x - 3)dx = \end{aligned}$$

La integral que nos ha quedado es del mismo tipo que la que pretendemos calcular, por lo que nuevamente aplicaremos el método de integración de partes:

Hacemos $\left\{ \begin{array}{l} u = 2x - 3 \Rightarrow du = 2dx \\ dv = e^x dx \Rightarrow v = e^x \end{array} \right\}$ y sustituimos:

$$\begin{aligned} I &= (x^2 - 3x + 5)e^x - \int e^x(2x - 3)dx = (x^2 - 3x + 5)e^x - \left[(2x - 3)e^x - \int 2e^x dx \right] = \\ &= (x^2 - 3x + 5)e^x - (2x - 3)e^x + 2 \int e^x dx = (x^2 - 3x + 5)e^x - (2x - 3)e^x + 2e^x + k = \\ &= [(x^2 - 3x + 5) - (2x - 3) + 2]e^x + k = e^x(x^2 - 5x + 10) + k \end{aligned}$$

$$\int x \ln(x) dx$$

$$\text{Sol: } \frac{1}{2}x^2 \left(\ln(x) - \frac{1}{2} \right) + k$$

Por el método de integración por partes:

$$\begin{aligned} \int x \ln(x) dx &= \left\{ \begin{array}{l} u = \ln(x) \Rightarrow du = \frac{1}{x} dx \\ dv = x dx \Rightarrow v = \frac{1}{2}x^2 \end{array} \right\} = \frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx = \\ &= \frac{1}{2}x^2 \ln(x) - \frac{1}{2} \int x dx = \frac{1}{2}x^2 \ln(x) - \frac{1}{2} \cdot \frac{1}{2}x^2 + k = \frac{1}{2}x^2 \left(\ln(x) - \frac{1}{2} \right) + k \end{aligned}$$

$$\int e^{-x} \cos x dx$$

$$\text{Sol: } \frac{1}{2} e^{-x} (\sin x - \cos x) + k$$

$$\begin{aligned} I &= \int e^{-x} \cos x dx = \left\{ \begin{array}{l} u = e^{-x} \Rightarrow du = -e^{-x} dx \\ dv = \cos x dx \Rightarrow v = \sin x \end{array} \right\} = e^{-x} \sin x - \int -e^{-x} \sin x dx = \\ &= e^{-x} \sin x + \int e^{-x} \sin x dx \end{aligned}$$

Al aplicar el método de partes nos ha quedado una integral del mismo tipo que la que pretendemos calcular, por lo que volvemos a aplicar el mismo método. En ella hacemos:

$$u = e^{-x} \Rightarrow du = -e^{-x} dx$$

$$dv = \sin x dx \Rightarrow v = -\cos x$$

Sustituyendo en la expresión anterior nos queda:

$$\begin{aligned} I &= e^{-x} \sin x + \int e^{-x} \sin x dx = e^{-x} \sin x + \left[-\cos x \cdot e^{-x} - \int -\cos x \cdot (-e^{-x}) dx \right] = \\ &= e^{-x} \sin x - \cos x \cdot e^{-x} - \int \cos x \cdot e^{-x} dx \end{aligned}$$

es decir, volvemos a la misma integral que pretendemos calcular. Entonces:

$$I = e^{-x} \sin x - \cos x \cdot e^{-x} - I \Rightarrow 2I = e^{-x} \sin x - \cos x \cdot e^{-x} \Rightarrow I = \frac{e^{-x} (\sin x - \cos x)}{2}$$

En consecuencia:

$$I = \int e^{-x} \cos x dx = \frac{e^{-x} (\sin x - \cos x)}{2} + k$$

$$\int \ln(1-x) dx$$

$$\text{Sol: } -x - (1-x) \ln(1-x) + k$$

$$\int \ln(1-x) dx = \left\{ \begin{array}{l} u = \ln(1-x) \Rightarrow du = \frac{-1}{1-x} dx \\ dv = dx \Rightarrow v = x \end{array} \right\} = x \ln(1-x) - \int x \cdot \frac{-1}{1-x} dx =$$

$$= x \ln(1-x) - \int \frac{-x}{1-x} dx = x \ln(1-x) - \int \frac{1-x-1}{1-x} dx = x \ln(1-x) - \int \left(1 + \frac{-1}{1-x} \right) dx =$$

$$= x \ln(1-x) - (x + \ln(1-x)) + k = x \ln(1-x) - x - \ln(1-x) + k =$$

$$= -x - (1-x) \ln(1-x) + k$$

$$\int x^n \mathbf{Ln}(x) dx \quad \text{Sol: } \frac{x^{n+1}}{n+1} \left(\mathbf{Ln}(x) - \frac{1}{n+1} \right) + k$$

Por el método de integración por partes:

$$\begin{aligned} \int x^n \mathbf{Ln}(x) dx &= \left\{ \begin{array}{l} u = \mathbf{Ln}(x) \Rightarrow du = \frac{1}{x} dx \\ dv = x^n dx \Rightarrow v = \frac{1}{n+1} x^{n+1} \end{array} \right\} = \frac{1}{n+1} x^{n+1} \mathbf{Ln}(x) - \int \frac{1}{n+1} x^{n+1} \cdot \frac{1}{x} dx = \\ &= \frac{1}{n+1} x^{n+1} \mathbf{Ln}(x) - \frac{1}{n+1} \int x^n dx = \frac{1}{n+1} x^{n+1} \mathbf{Ln}(x) - \frac{1}{n+1} \cdot \frac{1}{n+1} x^{n+1} + k = \\ &= \frac{x^{n+1}}{n+1} \left(\mathbf{Ln}(x) - \frac{1}{n+1} \right) + k \end{aligned}$$

$$\int \arcsen x dx \quad \text{Sol: } x \arcsen x + \sqrt{1-x^2} + k$$

Hacemos el siguiente cambio: $\left\{ \begin{array}{l} u = \arcsen x \\ dv = dx \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} du = \frac{1}{\sqrt{1-x^2}} \cdot dx \\ v = x \end{array} \right.$

Sustituyendo en la fórmula de integración por partes obtenemos:

$$\begin{aligned} \int \arcsen x dx &= x \cdot \arcsen x - \int x \cdot \frac{1}{\sqrt{1-x^2}} \cdot dx = x \cdot \arcsen x - \int x \cdot (1-x^2)^{-\frac{1}{2}} dx = \\ &= x \cdot \arcsen x + \frac{1}{2} \int -2x \cdot (1-x^2)^{-\frac{1}{2}} dx = x \cdot \arcsen x + \frac{1}{2} \cdot \frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + k = \\ &= x \cdot \arcsen x + \sqrt{1-x^2} + k \end{aligned}$$

$$\int \sqrt{1-x^2} dx \quad \text{Sol: } \frac{1}{2} (\arcsen x + x \sqrt{1-x^2}) + k$$

$$\begin{aligned} \int \sqrt{1-x^2} dx &= \int \frac{1-x^2}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} dx + \int \frac{-x^2}{\sqrt{1-x^2}} dx = \\ &= \arcsen x + \int \frac{-x^2}{\sqrt{1-x^2}} dx = \end{aligned}$$

La integral que nos queda la realizaremos por partes:

$$\int \frac{-x^2}{\sqrt{1-x^2}} dx = \int x \cdot \frac{-x}{\sqrt{1-x^2}} dx = \left\{ \begin{array}{l} u = x \Rightarrow du = dx \\ dv = \frac{-x}{\sqrt{1-x^2}} dx \Rightarrow v = \sqrt{1-x^2} \end{array} \right\} = x \sqrt{1-x^2} - \int \sqrt{1-x^2} dx$$

Sustituyendo nos queda:

$$\int \sqrt{1-x^2} dx = \arcsen x + \int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx = \arcsen x + x\sqrt{1-x^2} - \int \sqrt{1-x^2} dx$$

y se nos repite la misma integral. Entonces:

$$\begin{aligned} \int \sqrt{1-x^2} dx &= \arcsen x + x\sqrt{1-x^2} - \int \sqrt{1-x^2} dx \Rightarrow \\ \Rightarrow 2 \int \sqrt{1-x^2} dx &= \arcsen x + x\sqrt{1-x^2} \Rightarrow \int \sqrt{1-x^2} dx = \frac{1}{2} (\arcsen x + x\sqrt{1-x^2}) + k \end{aligned}$$

$$\int x \arcsen x dx \qquad \text{Sol: } \frac{1}{4} [(2x^2 - 1) \arcsen x + x\sqrt{1-x^2}] + k$$

Hacemos el siguiente cambio: $\begin{cases} u = \arcsen x \\ dv = x dx \end{cases} \Rightarrow \begin{cases} du = \frac{1}{\sqrt{1-x^2}} \cdot dx \\ v = \frac{x^2}{2} \end{cases}$

Sustituyendo en la fórmula de integración por partes obtenemos:

$$\int x \arcsen x \cdot dx = \frac{x^2}{2} \cdot \arcsen x - \int \frac{x^2}{2} \cdot \frac{1}{\sqrt{1-x^2}} \cdot dx = \frac{x^2}{2} \cdot \arcsen x + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx =$$

Por el ejercicio anterior tenemos que :

$$\begin{aligned} &= x\sqrt{1-x^2} - \int \sqrt{1-x^2} dx = x\sqrt{1-x^2} - \frac{1-x^2}{\sqrt{1-x^2}} dx \Rightarrow \\ \int \frac{-x^2}{\sqrt{1-x^2}} dx &= x\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{-x^2}{\sqrt{1-x^2}} dx = x\sqrt{1-x^2} - \arcsen x - \int \frac{-x^2}{\sqrt{1-x^2}} dx \end{aligned}$$

En consecuencia:

$$\int \frac{-x^2}{\sqrt{1-x^2}} dx = x\sqrt{1-x^2} - \arcsen x - \int \frac{-x^2}{\sqrt{1-x^2}} dx$$

Por tanto:

$$2 \int \frac{-x^2}{\sqrt{1-x^2}} dx = x\sqrt{1-x^2} - \arcsen x \Rightarrow \int \frac{-x^2}{\sqrt{1-x^2}} dx = \frac{1}{2} (x\sqrt{1-x^2} - \arcsen x)$$

Sustituyendo obtenemos:

$$\int x \arcsen x \cdot dx = \frac{x^2}{2} \cdot \arcsen x + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx =$$

$$\begin{aligned}
&= \frac{x^2}{2} \cdot \arcsin x + \frac{1}{2} \cdot \frac{1}{2} (x\sqrt{1-x^2} - \arcsin x) + k = \\
&= \frac{x^2}{2} \cdot \arcsin x + \frac{1}{4} x\sqrt{1-x^2} - \frac{1}{4} \arcsin x + k = \\
&= \frac{1}{4} [(2x^2 - 1)\arcsin x + x\sqrt{1-x^2}] + k
\end{aligned}$$

$$\int \arctan x \, dx \qquad \text{Sol: } x \arctan x - \frac{1}{2} \ln(1+x^2) + k$$

$$\begin{aligned}
\int \arctan x \, dx &= \left\{ \begin{array}{l} u = \arctan x \Rightarrow du = \frac{1}{1+x^2} dx \\ dv = dx \Rightarrow v = x \end{array} \right\} = x \cdot \arctan x - \int x \cdot \frac{1}{1+x^2} dx = \\
&= x \cdot \arctan x - \int \frac{x}{1+x^2} dx = x \cdot \arctan x - \frac{1}{2} \int \frac{2x}{1+x^2} dx = x \cdot \arctan x - \frac{1}{2} \ln(1+x^2) + k
\end{aligned}$$

$$\int \arctan \sqrt{x} \, dx \qquad \text{Sol: } (x+1) \arctan \sqrt{x} - \sqrt{x} + k$$

$$\begin{aligned}
\int \arctan \sqrt{x} \, dx &= \left\{ \begin{array}{l} u = \arctan \sqrt{x} \Rightarrow du = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} dx \\ dv = dx \Rightarrow v = x \end{array} \right\} = x \arctan \sqrt{x} - \int x \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} dx = \\
&= x \arctan \sqrt{x} - \frac{1}{2} \int \frac{\sqrt{x}}{1+x} dx = \left\{ \begin{array}{l} x = t^2 \Rightarrow \\ \Rightarrow dx = 2t dt \end{array} \right\} = x \arctan \sqrt{x} - \frac{1}{2} \int \frac{t}{1+t^2} \cdot 2t dt = \\
&= x \arctan \sqrt{x} - \int \frac{t^2}{1+t^2} dt = x \arctan \sqrt{x} - \int \frac{1+t^2-1}{1+t^2} dt = \\
&= x \arctan \sqrt{x} - \int \frac{1+t^2-1}{1+t^2} dt = x \arctan \sqrt{x} - \int \left(1 - \frac{1}{1+t^2} \right) dt = \\
&= x \arctan \sqrt{x} - \int dt + \int \frac{1}{1+t^2} dt = x \arctan \sqrt{x} - t + \arctan t + k = \\
&= x \arctan \sqrt{x} - \sqrt{x} + \arctan \sqrt{x} + k = (x+1) \arctan \sqrt{x} - \sqrt{x} + k
\end{aligned}$$

$$\int \mathbf{Ln} (x + \sqrt{1+x^2}) dx$$

$$\text{Sol: } x \mathbf{Ln} (x + \sqrt{1+x^2}) - \sqrt{1+x^2} + k$$

Hacemos: $u = \mathbf{Ln} (x + \sqrt{1+x^2})$ y $dv = dx$ con lo cual

$$du = \frac{1}{x + \sqrt{1+x^2}} \cdot \left(1 + \frac{2x}{2\sqrt{1+x^2}} \right) dx = \frac{1}{x + \sqrt{1+x^2}} \cdot \left(1 + \frac{x}{\sqrt{1+x^2}} \right) dx \Rightarrow$$

$$\Rightarrow du = \frac{1}{x + \sqrt{1+x^2}} \cdot \left(\frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}} \right) dx = \frac{1}{\sqrt{1+x^2}} dx \quad y \quad v = x$$

Sustituyendo en la fórmula de integración por partes, obtenemos:

$$\begin{aligned} \int \mathbf{Ln} (x + \sqrt{1+x^2}) dx &= x \cdot \mathbf{Ln} (x + \sqrt{1+x^2}) - \int x \cdot \frac{1}{\sqrt{1+x^2}} dx = \\ &= x \cdot \mathbf{Ln} (x + \sqrt{1+x^2}) - \int \frac{2x}{2\sqrt{1+x^2}} dx = x \cdot \mathbf{Ln} (x + \sqrt{1+x^2}) - \sqrt{1+x^2} + k \end{aligned}$$

$$\int \frac{x \arcsen x}{\sqrt{1-x^2}} dx$$

$$\text{Sol: } x - \sqrt{1-x^2} \arcsen x + k$$

$$\int \frac{x \arcsen x}{\sqrt{1-x^2}} dx = \left\{ \begin{array}{l} u = \arcsen x \Rightarrow du = \frac{1}{\sqrt{1-x^2}} dx \\ dv = \frac{x}{\sqrt{1-x^2}} dx \Rightarrow v = -\int \frac{-2x}{2\sqrt{1-x^2}} dx = -\sqrt{1-x^2} \end{array} \right\} =$$

$$= -\sqrt{1-x^2} \cdot \arcsen x - \int -\sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} \cdot \arcsen x + \int dx =$$

$$= -\sqrt{1-x^2} \cdot \arcsen x + x + k$$