

Calcular el MCD y el mcm de:

1. $P(x) = 2x^4 - 3x^3 + x$, $Q(x) = 2x^4 + 3x^3 - x^2 - 3x - 1$

2. $P(x) = x^5 - x^4 - x^3 + x^2$, $Q(x) = 2x^5 - 3x^4 + x^3$

Solución:

1. $P(x) = 2x^4 - 3x^3 + x$, $Q(x) = 2x^4 + 3x^3 - x^2 - 3x - 1$

$$P(x) = 2x^4 - 3x^3 + x = x(x-1)^2(2x+1)$$

$$Q(x) = 2x^4 + 3x^3 - x^2 - 3x - 1 = (x-1)(x+1)^2(2x+1)$$

$$\text{MCD}(P(x), Q(x)) = (x-1)(2x-1)$$

$$\text{mcm}(P(x), Q(x)) = x(x-1)^2(x+1)^2(2x+1)$$

2. $P(x) = x^5 - x^4 - x^3 + x^2$, $Q(x) = 2x^5 - 3x^4 + x^3$

$$P(x) = x^5 - x^4 - x^3 + x^2 = x^2(x+1)(x-1)^2$$

$$Q(x) = 2x^5 - 3x^4 + x^3 = x^3(x-1)(2x-1)$$

$$\text{MCD}(P(x), Q(x)) = x^2(x-1)$$

$$\text{mcm}(P(x), Q(x)) = x^3(x-1)^2(x+1)(2x-1)$$

Calcular el MCD y el mcm de:

1. $P(x) = 2x^4 + 5x^3 + 4x^2 + x$, $Q(x) = 2x^4 - x^3 - 3x^2 + x + 1$

2. $P(x) = x^5 + x^4 - x^3 - x^2$, $Q(x) = 2x^5 + x^4 - x^3$

Solución:

1. $P(x) = 2x^4 + 5x^3 + 4x^2 + x$, $Q(x) = 2x^4 - x^3 - 3x^2 + x + 1$

$$P(x) = 2x^4 + 5x^3 + 4x^2 + x = x(x+1)^2(2x+1)$$

$$Q(x) = 2x^4 - x^3 - 3x^2 + x + 1 = (x+1)(x-1)^2(2x+1)$$

$$\text{MCD}(P(x), Q(x)) = (x+1)(2x+1)$$

$$\text{mcm}(P(x), Q(x)) = x(x-1)^2(x+1)^2(2x+1)$$

2. $P(x) = x^5 + x^4 - x^3 - x^2$, $Q(x) = 2x^5 + x^4 - x^3$

$$P(x) = x^5 + x^4 - x^3 - x^2 = x^2(x-1)(x+1)^2$$

$$Q(x) = 2x^5 + x^4 - x^3 = x^3(x+1)(2x-1)$$

$$\text{MCD}(P(x), Q(x)) = x^2(x+1)$$

$$\text{mcm}(P(x), Q(x)) = x^3(x-1)(x+1)^2(2x-1)$$

Calcula el MCD y el mcm de los siguientes polinomios

$$P(x) = x^5 + x^4 - 7x^3 + x^2 + 10x - 6$$

$$Q(x) = x^5 + 5x^4 + x^3 - 19x^2 - 6x + 18$$

Solución: $P(x) = x^5 + x^4 - 7x^3 + x^2 + 10x - 6 = (x-1)^2(x+3)(x^2-2)$

$$Q(x) = x^5 + 5x^4 + x^3 - 19x^2 - 6x + 18 = (x-1)(x+3)^2(x^2-2)$$

$$\text{MCD}(P(x), Q(x)) = (x-1)(x+3)(x^2-2) = x^4 + 2x^3 - 5x^2 - 4x + 6$$

$$\begin{aligned} \text{mcm}(P(x), Q(x)) &= (x-1)^2(x+3)^2(x^2-2) = \\ &= x^6 + 4x^5 - 4x^4 - 20x^3 + 13x^2 + 24x - 18 \end{aligned}$$

Calcular el MCD y el mcm de:

1. $P(x) = x^4 + 4x^3 + 5x^2 + 2x$, $Q(x) = x^5 - x^4 - x^3 + x^2$
2. $P(x) = x^4 + 3x^3 - 4x$, $Q(x) = x^4 - 3x^2 + 2x$

Solución:

1. $P(x) = x^4 + 4x^3 + 5x^2 + 2x$, $Q(x) = x^5 - x^4 - x^3 + x^2$

$$\begin{aligned} P(x) &= x^4 + 4x^3 + 5x^2 + 2x = x(x+1)^2(x+2) \\ Q(x) &= x^5 - x^4 - x^3 + x^2 = x^2(x-1)^2(x+1) \end{aligned}$$

$$\text{MCD}(P(x), Q(x)) = x(x+1)$$

$$\text{mcm}(P(x), Q(x)) = x^2(x-1)^2(x+1)^2(x+2)$$

2. $P(x) = x^4 + 3x^3 - 4x$, $Q(x) = x^4 - 3x^2 + 2x$

$$P(x) = x^4 + 3x^3 - 4x = x(x+2)^2(x-1)$$

$$Q(x) = x^4 - 3x^2 + 2x = x(x-1)^2(x+2)$$

$$\text{MCD}(P(x), Q(x)) = x(x+2)(x-1)$$

$$\text{mcm}(P(x), Q(x)) = x(x+2)^2(x-1)^2$$

Calcular el MCD y el mcm de:

1. $P(x) = 2x^4 - 5x^3 + 4x^2 - x$, $Q(x) = 2x^4 + x^3 - 3x^2 - x + 1$
2. $P(x) = x^5 - 3x^3 + 2x^2$, $Q(x) = 2x^5 - 3x^4 + x^3$

Solución:

1. $P(x) = 2x^4 - 5x^3 + 4x^2 - x$, $Q(x) = 2x^4 + x^3 - 3x^2 - x + 1$

$$\begin{aligned} P(x) &= 2x^4 - 5x^3 + 4x^2 - x = x(x-1)^2(2x-1) \\ Q(x) &= 2x^4 + x^3 - 3x^2 - x + 1 = (x-1)(x+1)^2(2x-1) \end{aligned}$$

$$\text{MCD}(P(x), Q(x)) = (x-1)(2x-1)$$

$$\text{mcm}(P(x), Q(x)) = x(x-1)^2(x+1)^2(2x-1)$$

2. $P(x) = x^5 - 3x^3 + 2x^2$, $Q(x) = 2x^5 - 3x^4 + x^3$

$$\begin{aligned} P(x) &= x^5 - 3x^3 + 2x^2 = x^2(x+2)(x-1)^2 \\ Q(x) &= 2x^5 - 3x^4 + x^3 = x^3(x-1)(2x-1) \end{aligned}$$

$$\text{MCD}(P(x), Q(x)) = x^2(x-1)$$

$$\text{mcm}(P(x), Q(x)) = x^3(x-1)^2(x+2)(2x-1)$$

Calcular el MCD y el mcm de:

1. $P(x) = x^4 - 4x^3 + 5x^2 - 2x$, $Q(x) = x^5 - x^4 - x^3 + x^2$
2. $P(x) = 2x^5 + 5x^4 + 3x^3 - x^2 - x$, $Q(x) = 2x^4 + x^3 - 3x^2 - x + 1$

Solución:

$$1. P(x) = x^4 - 4x^3 + 5x^2 - 2x, Q(x) = x^5 - x^4 - x^3 + x^2$$

$$P(x) = x^4 - 4x^3 + 5x^2 - 2x = x(x-1)^2(x-2)$$

$$Q(x) = x^5 - x^4 - x^3 + x^2 = x^2(x-1)^2(x+1)$$

$$\text{MCD}(P(x), Q(x)) = x(x-1)^2$$

$$\text{mcm}(P(x), Q(x)) = x^2(x-1)^2(x+1)(x-2)$$

$$2. P(x) = 2x^5 + 5x^4 + 3x^3 - x^2 - x, Q(x) = 2x^4 + x^3 - 3x^2 - x + 1$$

$$P(x) = 2x^5 + 5x^4 + 3x^3 - x^2 - x = x(x+1)^3(2x-1)$$

$$Q(x) = 2x^4 + x^3 - 3x^2 - x + 1 = (x+1)^2(x-1)(2x-1)$$

$$\text{MCD}(P(x), Q(x)) = (x+1)^2(2x-1)$$

$$\text{mcm}(P(x), Q(x)) = x(x+1)^3(x-1)(2x-1)$$

Calcular el MCD y el mcm de:

$$1. P(x) = x^4 - x^3 - 5x^2 - 3x, Q(x) = x^5 - x^4 - x^3 + x^2$$

$$2. P(x) = 3x^5 - 10x^4 + 12x^3 - 6x^2 + x, Q(x) = 3x^4 - 4x^3 - 2x^2 + 4x - 1$$

Solución:

$$1. x^4 - x^3 - 5x^2 - 3x, Q(x) = x^5 - x^4 - x^3 + x^2$$

$$P(x) = x^4 - x^3 - 5x^2 - 3x = x(x+1)^2(x-3)$$

$$Q(x) = x^5 - x^4 - x^3 + x^2 = x^2(x-1)^2(x+1)$$

$$\text{MCD}(P(x), Q(x)) = x(x+1)$$

$$\text{mcm}(P(x), Q(x)) = x^2(x+1)^2(x-1)^2(x-3)$$

$$2. P(x) = 3x^5 - 10x^4 + 12x^3 - 6x^2 + x, Q(x) = 3x^4 - 4x^3 - 2x^2 + 4x - 1$$

$$P(x) = 3x^5 - 10x^4 + 12x^3 - 6x^2 + x = x(x-1)^3(3x-1)$$

$$Q(x) = 3x^4 - 4x^3 - 2x^2 + 4x - 1 = (x-1)^2(x+1)(3x-1)$$

$$\text{MCD}(P(x), Q(x)) = (x-1)^2(3x-1)$$

$$\text{mcm}(P(x), Q(x)) = x(x-1)^3(x+1)(3x-1)$$

Calcular el MCD y el mcm de:

$$1. P(x) = x^4 - 7x^3 + 11x^2 - 5x, Q(x) = x^5 + x^4 - x^3 - x^2$$

$$2. P(x) = 3x^4 + 7x^3 + 5x^2 + x, Q(x) = 3x^5 + 7x^4 + 2x^3 - 6x^2 - 5x - 1$$

Solución:

$$1. P(x) = x^4 - 7x^3 + 11x^2 - 5x, Q(x) = x^5 + x^4 - x^3 - x^2$$

$$P(x) = x^4 - 7x^3 + 11x^2 - 5x = x(x-1)^2(x-5)$$

$$Q(x) = x^5 + x^4 - x^3 - x^2 = x^2(x+1)^2(x-1)$$

$$\text{MCD}(P(x), Q(x)) = x(x-1)$$

$$\text{mcm}(P(x), Q(x)) = x^2(x+1)^2(x-1)^2(x-5)$$

$$2. P(x) = 3x^4 + 7x^3 + 5x^2 + x, Q(x) = 3x^5 + 7x^4 + 2x^3 - 6x^2 - 5x - 1$$

$$P(x) = 3x^4 + 7x^3 + 5x^2 + x = x(x+1)^2(3x+1)$$

$$Q(x) = 3x^5 + 7x^4 + 2x^3 - 6x^2 - 5x - 1 = (x+1)^3(x-1)(3x+1)$$

$$\text{MCD}(P(x), Q(x)) = (x+1)^2(3x+1)$$

$$\text{mcm}(P(x), Q(x)) = x(x+1)^3(x-1)(3x+1)$$

Si $P(x) = (x-3)^2x^2$, busca un polinomio de tercer grado, $Q(x)$, que cumpla las dos condiciones siguientes:

$$1. \text{MCD}(P(x), Q(x)) = x^2 - 3x = x(x-3)$$

$$2. \text{mcm}(P(x); Q(x)) = (x-3)^2x^2(x+7)$$

Solución: $P(x) = x(x-3)(x+7) = x^3 + 4x^2 - 21x$

Si $P(x) = (x+3)^2x^2$, busca un polinomio de tercer grado, $Q(x)$, que cumpla las dos condiciones siguientes:

$$1. \text{MCD}(P(x), Q(x)) = x^2 + 3x = x(x+3)$$

$$2. \text{mcm}(P(x); Q(x)) = (x+3)^2x^2(x-7)$$

Solución: $P(x) = x(x+3)(x-7) = x^3 - 4x^2 - 21x$

Problema 210 Si $P(x) = (x-2)^3x^2$, busca un polinomio de tercer grado, $Q(x)$, que cumpla las dos condiciones siguientes:

$$1. \text{MCD}(P(x), Q(x)) = x(x-2)^2$$

$$2. \text{mcm}(P(x); Q(x)) = (x-2)^3x^2(x+1)$$

Solución: $Q(x) = x(x-2)^2(x+1) = x^4 - 3x^3 + 4x$

Si $P(x) = (x-5)^2x^2$, busca un polinomio de tercer grado, $Q(x)$, que cumpla las dos condiciones siguientes:

$$1. \text{MCD}(P(x), Q(x)) = x^2 - 5x = x(x-5)$$

$$2. \text{mcm}(P(x); Q(x)) = (x-5)^2x^2(x+6)$$

Solución: $P(x) = x(x-5)(x+6) = x^3 + x^2 - 30x$

Si $P(x) = (x-6)^2x^2$, busca un polinomio de tercer grado, $Q(x)$, que cumpla las dos condiciones siguientes:

$$1. \text{MCD}(P(x), Q(x)) = x^2 - 6x = x(x-6)$$

$$2. \text{mcm}(P(x); Q(x)) = (x-6)^2x^2(x+5)$$

Solución: $P(x) = x(x+5)(x-6) = x^3 - x^2 - 30x$

Problema 213 Si $P(x) = (x-7)^2x^2$, busca un polinomio de tercer grado, $Q(x)$, que cumpla las dos condiciones siguientes:

$$1. \text{MCD}(P(x), Q(x)) = x^2 - 7x = x(x-7)$$

$$2. \text{mcm}(P(x); Q(x)) = (x-7)^2x^2(x+1)$$

Solución: $P(x) = x(x+1)(x-7) = x^3 - 6x^2 - 7x$