

INTEGRALES ELEMENTALES

El Propósito de este capítulo, antes de conocer y practicar las técnicas propiamente tales; es familiarizarse con aquellas integrales para las cuales basta una transformación algebraica elemental.

EJERCICIOS DESARROLLADOS

1.1.- Encontrar: $\int e^{\ell\eta x^2} x dx$

Solución.- Se sabe que: $e^{\ell\eta x^2} = x^2$

Por lo tanto: $\int e^{\ell\eta x^2} x dx = \int x^2 x dx = \int x^3 dx = \frac{x^4}{4} + c$

Respuesta: $\int e^{\ell\eta x^2} x dx = \frac{x^4}{4} + c$, Fórmula utilizada: $\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$

1.2.- Encontrar: $\int 3a^7 x^6 dx$

Solución.-

$$\int 3a^7 x^6 dx = 3a^7 \int x^6 dx = 3a^7 \frac{x^7}{7} + c$$

Respuesta: $\int 3a^7 x^6 dx = 3a^7 \frac{x^7}{7} + c$, Fórmula utilizada: del ejercicio anterior.

1.3.- Encontrar: $\int (3x^2 + 2x + 1) dx$

Solución.-

$$\begin{aligned} \int (3x^2 + 2x + 1) dx &= \int (3x^2 + 2x + 1) dx = \int 3x^2 dx + \int 2x dx + \int dx \\ &= 3 \int x^2 dx + 2 \int x dx + \int dx = \cancel{\beta} \frac{x^3}{\cancel{\beta}} + \cancel{\cancel{Z}} \frac{x^2}{\cancel{Z}} + x + c = x^3 + x^2 + x + c \end{aligned}$$

Respuesta: $\int (3x^2 + 2x + 1) dx = x^3 + x^2 + x + c$

1.4.- Encontrar: $\int x(x+a)(x+b) dx$

Solución.-

$$\begin{aligned} \int x(x+a)(x+b) dx &= \int x [x^2 + (a+b)x + ab] dx = \int [x^3 + (a+b)x^2 + abx] dx \\ &= \int x^3 dx + \int (a+b)x^2 dx + \int abx dx = \int x^3 dx + (a+b) \int x^2 dx + ab \int x dx \\ &= \frac{x^4}{4} + (a+b) \frac{x^3}{3} + ab \frac{x^2}{2} + c \end{aligned}$$

Respuesta: $\int x(x+a)(x+b) dx = \frac{x^4}{4} + \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c$

1.5.- Encontrar: $\int (a+bx^3)^2 dx$

Solución.-

$$\begin{aligned} \int (a+bx^3)^2 dx &= \int (a^2 + 2abx^3 + b^2x^6) dx = \int a^2 dx + \int 2abx^3 dx + \int b^2x^6 dx \\ &= a^2 \int dx + 2ab \int x^3 dx + b^2 \int x^6 dx = a^2 x + 2ab \frac{x^4}{4} + b^2 \frac{x^7}{7} + c \end{aligned}$$

Respuesta: $\int (a+bx^3)^2 dx = a^2 x + \frac{abx^4}{2} + \frac{b^2x^7}{7} + c$

1.6.- Encontrar: $\int \sqrt{2px} dx$

Solución.-

$$\int \sqrt{2px} dx = \int \sqrt{2px^{\frac{1}{2}}} dx = \sqrt{2p} \int x^{\frac{1}{2}} dx = \sqrt{2p} \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2\sqrt{2px^{\frac{3}{2}}}}{3} + c$$

Respuesta: $\int \sqrt{2px} dx = \frac{2\sqrt{2px}\sqrt{x}}{3} + c$

1.7.- Encontrar: $\int \frac{dx}{\sqrt[n]{x}}$

Solución.-

$$\int \frac{dx}{\sqrt[n]{x}} = \int x^{-\frac{1}{n}} dx = \frac{x^{-\frac{1}{n}+1}}{-\frac{1}{n}+1} + c = \frac{x^{\frac{-1+n}{n}}}{\frac{-1+n}{n}} + c = \frac{nx^{\frac{-1+n}{n}}}{n-1} + c$$

Respuesta: $\int \frac{dx}{\sqrt[n]{x}} = \frac{nx^{\frac{-1+n}{n}}}{n-1} + c$

1.8.- Encontrar: $\int (nx)^{\frac{1-n}{n}} dx$

Solución.-

$$\begin{aligned} \int (nx)^{\frac{1-n}{n}} dx &= \int n^{\frac{1-n}{n}} x^{\frac{1-n}{n}} dx = n^{\frac{1-n}{n}} \int x^{\frac{1-n}{n}} dx = n^{\frac{1-n}{n}} \int x^{\frac{1}{n}-1} dx \\ &= n^{\frac{1-n}{n}} \frac{x^{\frac{1}{n}-1+1}}{\frac{1}{n}-1+1} + c = n^{\frac{1-n}{n}} \frac{x^{\frac{1}{n}}}{\frac{1}{n}} + c = n^{\frac{1-n}{n}} nx^{\frac{1}{n}} + c = n^{\frac{1-n}{n}+1} x^{\frac{1}{n}} + c = n^{\frac{1-n+n}{n}} x^{\frac{1}{n}} + c = n^{\frac{1}{n}} x^{\frac{1}{n}} + c \end{aligned}$$

Respuesta: $\int (nx)^{\frac{1-n}{n}} dx = \sqrt[n]{nx} + c$

1.9.- Encontrar: $\int (a^{\frac{2}{3}} - x^{\frac{2}{3}})^3 dx$

Solución.-

$$\begin{aligned} \int (a^{\frac{2}{3}} - x^{\frac{2}{3}})^3 dx &= \int \left[(a^{\frac{2}{3}})^3 - 3(a^{\frac{2}{3}})^2 x^{\frac{2}{3}} + 3a^{\frac{2}{3}} (x^{\frac{2}{3}})^2 - (x^{\frac{2}{3}})^3 \right] dx \\ &= \int (a^2 - 3a^{\frac{4}{3}} x^{\frac{2}{3}} + 3a^{\frac{2}{3}} x^{\frac{4}{3}} - x^2) dx = \int a^2 dx - \int 3a^{\frac{4}{3}} x^{\frac{2}{3}} dx + \int 3a^{\frac{2}{3}} x^{\frac{4}{3}} dx - \int x^2 dx \\ &= a^2 \int dx - 3a^{\frac{4}{3}} \int x^{\frac{2}{3}} dx + 3a^{\frac{2}{3}} \int x^{\frac{4}{3}} dx - \int x^2 dx = a^2 x - 3a^{\frac{4}{3}} \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 3a^{\frac{2}{3}} \frac{x^{\frac{7}{3}}}{\frac{7}{3}} - \frac{x^3}{3} + c \\ &= a^2 x - \frac{9a^{\frac{4}{3}} x^{\frac{5}{3}}}{5} + \frac{9a^{\frac{2}{3}} x^{\frac{7}{3}}}{7} - \frac{x^3}{3} + c \end{aligned}$$

Respuesta: $\int (a^{\frac{2}{3}} - x^{\frac{2}{3}})^3 dx = a^2 x - \frac{9a^{\frac{4}{3}} x^{\frac{5}{3}}}{5} + \frac{9a^{\frac{2}{3}} x^{\frac{7}{3}}}{7} - \frac{x^3}{3} + c$

1.10.- Encontrar: $\int (\sqrt{x}+1)(x-\sqrt{x}+1) dx$

Solución.-

$$\begin{aligned} \int (\sqrt{x}+1)(x-\sqrt{x}+1) dx &= (x\sqrt{x} - \cancel{(\sqrt{x})^2} + \cancel{\sqrt{x}} + \cancel{x} - \cancel{\sqrt{x}} + 1) dx \\ &= \int (x\sqrt{x}+1) dx = \int (xx^{\frac{1}{2}}+1) dx = \int (x^{\frac{3}{2}}+1) dx = \int x^{\frac{3}{2}} dx + \int dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + x + c = \frac{2x^{\frac{5}{2}}}{5} + x + c \end{aligned}$$

Respuesta: $\int (\sqrt{x}+1)(x-\sqrt{x}+1) dx = \frac{2x^{\frac{5}{2}}}{5} + x + c$

1.11.- Encontrar: $\int \frac{(x^2+1)(x^2-2) dx}{\sqrt[3]{x^2}}$

Solución.-

$$\begin{aligned} \int \frac{(x^2+1)(x^2-2)dx}{\sqrt[3]{x^2}} &= \int \frac{(x^4-x^2-2)dx}{x^{2/3}} = \int \frac{x^4}{x^{2/3}} dx - \int \frac{x^2}{x^{2/3}} dx - \int \frac{2}{x^{2/3}} dx \\ &= \int x^{10/3} dx - \int x^{4/3} dx - 2 \int x^{-2/3} dx = \frac{x^{10/3+1}}{\frac{10}{3}+1} - \frac{x^{4/3+1}}{\frac{4}{3}+1} - 2 \frac{x^{-2/3+1}}{-2/3+1} = \frac{x^{13/3}}{\frac{13}{3}} - \frac{x^{7/3}}{\frac{7}{3}} - 2 \frac{x^{1/3}}{1/3} + c \\ &= 3 \frac{x^{13/3}}{13} - 3 \frac{x^{7/3}}{7} - 6x^{1/3} + c = 3 \frac{\sqrt[3]{x^{13}}}{13} - 3 \frac{\sqrt[3]{x^7}}{7} - 6\sqrt[3]{x} + c = 3 \frac{x^4 \sqrt[3]{x}}{13} - 3 \frac{x^2 \sqrt[3]{x}}{7} - 6\sqrt[3]{x} + c \end{aligned}$$

Respuesta: $\int \frac{(x^2+1)(x^2-2)dx}{\sqrt[3]{x^2}} = \left(\frac{3x^4}{13} - \frac{3x^2}{7} - 6 \right) \sqrt[3]{x} + c$

1.12.- Encontrar: $\int \frac{(x^m - x^n)^2}{\sqrt{x}} dx$

Solución.-

$$\begin{aligned} \int \frac{(x^m - x^n)^2}{\sqrt{x}} dx &= \int \frac{(x^{2m} - 2x^m x^n + x^{2n})}{\sqrt{x}} dx = \int \frac{(x^{2m} - 2x^m x^n + x^{2n})}{x^{1/2}} dx \\ &= \int (x^{2m-1/2} - 2x^{m+n-1/2} + x^{2n-1/2}) dx = \frac{x^{2m-1/2+1}}{2m-1/2+1} - \frac{2x^{m+n+1/2}}{m+n+1/2} + \frac{x^{2n+1/2}}{2n+1/2} + c \\ &= \frac{x^{\frac{4m+1}{2}}}{\frac{4m+1}{2}} - \frac{2x^{\frac{2m+2n+1}{2}}}{\frac{2m+2n+1}{2}} + \frac{x^{\frac{4n+1}{2}}}{\frac{4n+1}{2}} + c = \frac{2x^{\frac{4m+1}{2}}}{4m+1} - \frac{4x^{\frac{2m+2n+1}{2}}}{2m+2n+1} + \frac{2x^{\frac{4n+1}{2}}}{4n+1} + c \\ &= \frac{2x^{2m} \sqrt{x}}{4m+1} - \frac{4x^{m+n} \sqrt{x}}{2m+2n+1} + \frac{2x^{2n} \sqrt{x}}{4n+1} + c \end{aligned}$$

Respuesta: $\int \frac{(x^m - x^n)^2}{\sqrt{x}} dx = \sqrt{x} \left(\frac{2x^{2m}}{4m+1} - \frac{4x^{m+n}}{2m+2n+1} + \frac{2x^{2n}}{4n+1} \right) + c$

1.13.- Encontrar: $\int \frac{(\sqrt{a} - \sqrt{x})^4}{\sqrt{ax}} dx$

Solución.-

$$\begin{aligned} \int \frac{(\sqrt{a} - \sqrt{x})^4}{\sqrt{ax}} dx &= \int \frac{a^2 - 4a\sqrt{ax} + 6xa - 4x\sqrt{ax} + x^2}{\sqrt{ax}} dx \\ &= \int \frac{a^2}{(ax)^{1/2}} dx - \int \frac{4a\sqrt{ax}}{\sqrt{ax}} dx + \int \frac{6ax}{(ax)^{1/2}} dx - \int \frac{4x\sqrt{ax}}{\sqrt{ax}} dx + \int \frac{x^2}{(ax)^{1/2}} dx \\ &= \int a^2 a^{-1/2} x^{-1/2} dx - \int 4adx + \int 6aa^{-1/2} xx^{-1/2} dx - \int 4x dx + \int a^{-1/2} x^2 x^{-1/2} dx \\ &= a^{3/2} \int x^{-1/2} dx - 4a \int dx + 6a^{1/2} \int x^{1/2} dx - 4 \int x dx + a^{-1/2} \int x^{3/2} dx \\ &= a^{3/2} \frac{x^{-1/2+1}}{-1/2+1} - 4ax + 6a^{1/2} \frac{x^{1/2+1}}{1/2+1} - 4 \frac{x^{1+1}}{1+1} + a^{-1/2} \frac{x^{3/2+1}}{3/2+1} + c \\ &= a^{3/2} \frac{x^{1/2}}{1/2} - 4ax + 6a^{1/2} \frac{x^{3/2}}{3/2} - 4 \frac{x^2}{2} + a^{-1/2} \frac{x^{5/2}}{5/2} + c \\ &= 2a^{3/2} x^{1/2} - 4ax + 4a^{1/2} x^{3/2} - 2x^2 + 2a^{-1/2} \frac{x^{5/2}}{5} + c \end{aligned}$$

Respuesta: $\int \frac{(\sqrt{a}-\sqrt{x})^4}{\sqrt{ax}} dx = 2a^{\frac{3}{2}}x^{\frac{1}{2}} - 4ax + 4a^{\frac{1}{2}}x^{\frac{3}{2}} - 2x^2 + \frac{2x^3}{5\sqrt{xa}} + c$

1.14.- Encontrar: $\int \frac{dx}{x^2-10}$

Solución.-

Sea: $a = \sqrt{10}$, Luego: $\int \frac{dx}{x^2-10} = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ell \eta \left| \frac{x-a}{x+a} \right| + c$
 $= \frac{1}{2\sqrt{10}} \ell \eta \left| \frac{x-\sqrt{10}}{x+\sqrt{10}} \right| + c = \frac{\sqrt{10}}{20} \ell \eta \left| \frac{x-\sqrt{10}}{x+\sqrt{10}} \right| + c$

Respuesta: $\int \frac{dx}{x^2-10} = \frac{\sqrt{10}}{20} \ell \eta \left| \frac{x-\sqrt{10}}{x+\sqrt{10}} \right| + c$

1.15.- Encontrar: $\int \frac{dx}{x^2+7}$

Solución.- Sea: $a = \sqrt{7}$, Luego: $\int \frac{dx}{x^2+7} = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \text{arc } \tau g \frac{x}{a} + c$

$\frac{1}{\sqrt{7}} \text{arc } \tau g \frac{x}{\sqrt{7}} + c = \frac{\sqrt{7}}{7} \text{arc } \tau g \frac{\sqrt{7}x}{a} + c$

Respuesta: $\int \frac{dx}{x^2+7} = \frac{\sqrt{7}}{7} \text{arc } \tau g \frac{\sqrt{7}x}{a} + c$

1.16.- Encontrar: $\int \sqrt{\frac{dx}{4+x^2}}$

Solución.-

Sea: $a = 2$, Luego: $\int \frac{dx}{\sqrt{4+x^2}} = \int \frac{dx}{\sqrt{a^2+x^2}} = \ell \eta \left| x + \sqrt{a^2+x^2} \right| + c$
 $= \ell \eta \left| x + \sqrt{4+x^2} \right| + c$

Respuesta: $\int \frac{dx}{\sqrt{4+x^2}} = \ell \eta \left| x + \sqrt{4+x^2} \right| + c$

1.17.- Encontrar: $\int \frac{dx}{\sqrt{8-x^2}}$

Solución.-

Sea: $a = \sqrt{8}$, Luego: $\int \frac{dx}{\sqrt{8-x^2}} = \int \frac{dx}{\sqrt{a^2-x^2}} = \text{arcs } e n \frac{x}{a} + c$
 $= \text{arcs } e n \frac{x}{\sqrt{8}} + c = \text{arcs } e n \frac{x}{2\sqrt{2}} + c$

Respuesta: $\int \frac{dx}{\sqrt{8-x^2}} = \text{arcs } e n \frac{\sqrt{2}x}{4} + c$

1.18.- Encontrar: $\int \frac{dy}{x^2+9}$

Solución.-

La expresión: $\frac{1}{x^2+9}$ actúa como constante, luego:

$\int \frac{dy}{x^2+9} = \frac{1}{x^2+9} \int dy = \frac{1}{x^2+9} y + c = \frac{y}{x^2+9} + c$

Respuesta: $\int \frac{dy}{x^2+9} = \frac{y}{x^2+9} + c$

1.19.- Encontrar: $\int \frac{\sqrt{2+x^2} - \sqrt{2-x^2}}{\sqrt{4-x^4}} dx$

Solución.-

$$\begin{aligned} \int \frac{\sqrt{2+x^2} - \sqrt{2-x^2}}{\sqrt{4-x^4}} dx &= \int \sqrt{\frac{2+x^2}{4-x^4}} dx - \int \sqrt{\frac{2-x^2}{4-x^4}} dx \\ &= \int \sqrt{\frac{2+x^2}{(2-x^2)(2+x^2)}} dx - \int \sqrt{\frac{2-x^2}{(2-x^2)(2+x^2)}} dx = \int \frac{dx}{\sqrt{2-x^2}} - \int \frac{dx}{\sqrt{2+x^2}} \end{aligned}$$

Sea: $a = \sqrt{2}$, Luego: $\int \frac{dx}{\sqrt{a^2-x^2}} - \int \frac{dx}{\sqrt{a^2+x^2}} = \arcsen \frac{x}{a} - \ell\eta \left| x + \sqrt{a^2+x^2} \right| + c$
 $= \arcsen \frac{x}{\sqrt{2}} - \ell\eta \left| x + \sqrt{(\sqrt{2})^2+x^2} \right| + c = \arcsen \frac{x}{\sqrt{2}} - \ell\eta \left| x + \sqrt{2+x^2} \right| + c$

Respuesta: $\int \frac{\sqrt{2+x^2} - \sqrt{2-x^2}}{\sqrt{4-x^4}} dx = \arcsen \frac{x}{\sqrt{2}} - \ell\eta \left| x + \sqrt{2+x^2} \right| + c$

1.20.- Encontrar: $\int \tau g^2 x dx$

Solución.-

$$\int \tau g^2 x dx = \int (\sec^2 x - 1) dx = \int \sec^2 x dx - \int dx = \tau g x - x + c$$

Respuesta: $\int \tau g^2 x dx = \tau g x - x + c$

1.21.- Encontrar: $\int \text{co } \tau g^2 x dx$

Solución.-

$$\int \text{co } \tau g^2 x dx = \int (\text{cos } ec^2 x - 1) dx = \int \text{cos } ec^2 x dx - \int dx = -\text{co } \tau g x - x + c$$

Respuesta: $\int \text{co } \tau g^2 x dx = -\text{co } \tau g x - x + c$

1.22.- Encontrar: $\int \frac{dx}{2x^2+4}$

Solución.-

$$\int \frac{dx}{2x^2+4} = \int \frac{dx}{2(x^2+2)} = \frac{1}{2} \int \frac{dx}{x^2+2} = \frac{1}{2} \frac{1}{\sqrt{2}} \arctg \frac{x}{\sqrt{2}} + c = \frac{\sqrt{2}}{4} \arctg \frac{\sqrt{2}x}{2} + c$$

Respuesta: $\int \frac{dx}{2x^2+4} = \frac{\sqrt{2}}{4} \arctg \frac{\sqrt{2}x}{2} + c$

1.23.- Encontrar: $\int \frac{dx}{7x^2-8}$

Solución.-

$$\begin{aligned} \int \frac{dx}{7x^2-8} &= \int \frac{dx}{7(x^2-\frac{8}{7})} = \int \frac{dx}{7[(x^2-(\sqrt{\frac{8}{7}})^2)]} = \frac{1}{7} \int \frac{dx}{[x^2-(\sqrt{\frac{8}{7}})^2]} \\ &= \frac{1}{7} \frac{1}{2(\sqrt{\frac{8}{7}})} \ell\eta \left| \frac{x-\sqrt{\frac{8}{7}}}{x+\sqrt{\frac{8}{7}}} \right| + c = \frac{1}{14} \frac{\sqrt{8}}{\sqrt{7}} \ell\eta \left| \frac{x-\sqrt{\frac{8}{7}}}{x+\sqrt{\frac{8}{7}}} \right| + c = \frac{\sqrt{7}}{14\sqrt{8}} \ell\eta \left| \frac{\sqrt{7}x-\sqrt{8}}{\sqrt{7}x+\sqrt{8}} \right| + c \end{aligned}$$

$$= \frac{1}{4\sqrt{14}} \ell \eta \left| \frac{\sqrt{7x-2\sqrt{2}}}{\sqrt{7x+2\sqrt{2}}} \right| + c = \frac{\sqrt{14}}{56} \ell \eta \left| \frac{\sqrt{7x-2\sqrt{2}}}{\sqrt{7x+2\sqrt{2}}} \right| + c$$

Respuesta: $\int \frac{dx}{7x^2-8} = \frac{\sqrt{14}}{56} \ell \eta \left| \frac{\sqrt{7x-2\sqrt{2}}}{\sqrt{7x+2\sqrt{2}}} \right| + c$

1.24.- Encontrar: $\int \frac{x^2 dx}{x^2+3}$

Solución.-

$$\int \frac{x^2 dx}{x^2+3} = \int \left(1 - \frac{3}{x^2+3}\right) dx = \int dx - 3 \int \frac{dx}{x^2+3} = \int dx - 3 \int \frac{dx}{x^2+(\sqrt{3})^2}$$

$$= x - 3 \frac{1}{\sqrt{3}} \operatorname{arc} \tau g \frac{x}{\sqrt{3}} + c = x - \sqrt{3} \operatorname{arc} \tau g \frac{\sqrt{3}x}{3} + c$$

Respuesta: $\int \frac{x^2 dx}{x^2+3} = x - \sqrt{3} \operatorname{arc} \tau g \frac{\sqrt{3}x}{3} + c$

1.25.- Encontrar: $\int \frac{dx}{\sqrt{7+8x^2}}$

Solución.-

$$\int \frac{dx}{\sqrt{7+8x^2}} = \int \frac{dx}{\sqrt{(\sqrt{8}x)^2 + (\sqrt{7})^2}} = \frac{1}{\sqrt{8}} \ell \eta \left| \sqrt{8}x + \sqrt{7+8x^2} \right| + c$$

Respuesta: $\int \frac{dx}{\sqrt{7+8x^2}} = \frac{\sqrt{2}}{4} \ell \eta \left| \sqrt{8}x + \sqrt{7+8x^2} \right| + c$

1.26.- Encontrar: $\int \frac{dx}{\sqrt{7-5x^2}}$

Solución.-

$$\int \frac{dx}{\sqrt{7-5x^2}} = \int \frac{dx}{\sqrt{(\sqrt{7})^2 - (\sqrt{5}x)^2}} = \frac{1}{\sqrt{5}} \operatorname{arcs} e n x \frac{\sqrt{5}}{\sqrt{7}} + c$$

Respuesta: $\int \frac{dx}{\sqrt{7-5x^2}} = \frac{\sqrt{5}}{5} \operatorname{arcs} e n \frac{\sqrt{35}x}{7} + c$

1.27.- Encontrar: $\int \frac{(a^x - b^x)^2 dx}{a^x b^x}$

Solución.-

$$\int \frac{(a^x - b^x)^2 dx}{a^x b^x} = \int \frac{(a^{2x} - 2a^x b^x + b^{2x})}{a^x b^x} dx = \int \frac{a^{2x}}{a^x b^x} dx - \int \frac{2a^x b^x}{a^x b^x} dx + \int \frac{b^{2x}}{a^x b^x} dx$$

$$= \int \frac{a^x}{b^x} dx - \int 2dx + \int \frac{b^x}{a^x} dx = \int \left(\frac{a}{b}\right)^x dx - 2 \int dx + \int \left(\frac{b}{a}\right)^x dx = \frac{(a/b)^x}{\ell \eta \frac{a}{b}} - 2x + \frac{(b/a)^x}{\ell \eta \frac{b}{a}} + c$$

$$= \frac{(a/b)^x}{\ell \eta a - \ell \eta b} - 2x + \frac{(b/a)^x}{\ell \eta b - \ell \eta a} + c = \frac{(a/b)^x}{\ell \eta a - \ell \eta b} - 2x - \frac{(b/a)^x}{\ell \eta a - \ell \eta b} + c$$

$$= \frac{\left(\frac{a^x}{b^x} - \frac{b^x}{a^x}\right)}{\ell \eta a - \ell \eta b} - 2x + c$$

Respuesta: $\int \frac{(a^x - b^x)^2 dx}{a^x b^x} = \frac{\left(\frac{a^{2x} - b^{2x}}{a^x b^x}\right)}{\ell \eta a - \ell \eta b} - 2x + c$

1.28.- Encontrar: $\int \operatorname{sen}^2 \frac{x}{2} dx$

Solución.-

$$\int \operatorname{sen}^2 \frac{x}{2} dx = \int \frac{1 - \cos \cancel{\frac{x}{2}}}{2} dx = \int \frac{1 - \cos x}{2} dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos x dx$$

$$= \frac{x}{2} - \frac{\operatorname{sen} x}{2} + c$$

Respuesta: $\int \operatorname{sen}^2 \frac{x}{2} dx = \frac{x}{2} - \frac{\operatorname{sen} x}{2} + c$

1.29.- Encontrar: $\int \frac{dx}{(a+b) + (a-b)x^2}; (0 < b < a)$

Solución.-

Sea: $c^2 = a+b$, $d^2 = a-b$; luego $\int \frac{dx}{(a+b) + (a-b)x^2} = \int \frac{dx}{c^2 + d^2 x^2}$

$$\int \frac{dx}{d^2 \left(\frac{c^2}{d^2} + x^2 \right)} = \frac{1}{d^2} \int \frac{dx}{\left(\frac{c}{d} \right)^2 + x^2} = \frac{1}{d^2} \frac{1}{\cancel{d}} \operatorname{arctg} \frac{x}{\cancel{c/d}} + c = \frac{1}{cd} \operatorname{arctg} \frac{dx}{c} + c$$

$$= \frac{1}{\sqrt{a+b}\sqrt{a-b}} \operatorname{arctg} \frac{\sqrt{a-b}x}{\sqrt{a+b}} + c = \frac{1}{\sqrt{a^2-b^2}} \operatorname{arctg} \sqrt{\frac{a-b}{a+b}} x + c$$

Respuesta: $\int \frac{dx}{(a+b) + (a-b)x^2} = \frac{1}{\sqrt{a^2-b^2}} \operatorname{arctg} \sqrt{\frac{a-b}{a+b}} x + c$

1.30.- Encontrar: $\int \frac{dx}{(a+b) - (a-b)x^2}; (0 < b < a)$

Solución.-

Sea: $c^2 = a+b$, $d^2 = a-b$, Luego: $\int \frac{dx}{(a+b) - (a-b)x^2} = \int \frac{dx}{c^2 - d^2 x^2}$

$$= \int \frac{dx}{d^2 \left(\frac{c^2}{d^2} - x^2 \right)} = \frac{1}{d^2} \int \frac{dx}{\left(\frac{c}{d} \right)^2 - x^2} = -\frac{1}{d^2} \frac{1}{\cancel{2c}} \ell \eta \left| \frac{x - \cancel{c/d}}{x + \cancel{c/d}} \right| + c = -\frac{1}{2cd} \ell \eta \left| \frac{dx - c}{dx + c} \right| + c$$

$$= -\frac{1}{2\sqrt{a^2-b^2}} \ell \eta \left| \frac{\sqrt{a-b}x - \sqrt{a+b}}{\sqrt{a-b}x + \sqrt{a+b}} \right| + c$$

Respuesta: $\int \frac{dx}{(a+b) - (a-b)x^2} = -\frac{1}{2\sqrt{a^2-b^2}} \ell \eta \left| \frac{\sqrt{a-b}x - \sqrt{a+b}}{\sqrt{a-b}x + \sqrt{a+b}} \right| + c$

1.31.- Encontrar: $\int \left[(a^{2x})^0 - 1 \right] dx$

Solución.-

$$\int \left[(a^{2x})^0 - 1 \right] dx = \int (a^0 - 1) dx = \int (1 - 1) dx = \int dx - \int dx = \int 0 dx = c$$

Respuesta: $\int \left[(a^{2x})^0 - 1 \right] dx = c$

EJERCICIOS PROPUESTOS

Mediante el uso del álgebra elemental, o algunas identidades trigonométricas, transformar en integrales de fácil solución, las integrales que se presentan a continuación.

$$1.32.- \int 3x^5 dx$$

$$1.35.- \int \cos^2 \frac{x}{2} dx$$

$$1.38.- \int \frac{1 + \frac{\sqrt{x}}{2}}{1 + \frac{\sqrt{x}}{3}} dy$$

$$1.41.- \int \frac{dx}{\sqrt{x^2 + 5}}$$

$$1.44.- \int (\operatorname{sen}^2 x + \cos^2 x - 1) dx$$

$$1.47.- \int \frac{dx}{x^2 - 12}$$

$$1.50.- \int \frac{dx}{\sqrt{x^2 + 12}}$$

$$1.53.- \int \frac{dx}{x\sqrt{12 - x^2}}$$

$$1.56.- \int \frac{dx}{\sqrt{2x^2 - 8}}$$

$$1.59.- \int \sqrt{x^2 + 10} dx$$

$$1.62.- \int \sqrt{1 - \operatorname{sen}^2 x} dx$$

$$1.65.- \int (2^0 - 3^0)^n dx$$

$$1.68.- \int \sqrt{\frac{3}{4} - x^2} dx$$

$$1.71.- \int \frac{dx}{x\sqrt{3 - x^2}}$$

$$1.74.- \int \operatorname{sen}^{3x} \theta dy$$

$$1.77.- \int e^{\ell \eta x^2} dx$$

$$1.80.- \int \sqrt{x^2 - 11} dx$$

$$1.33.- \int (1 + e)^x dx$$

$$1.36.- \int (1 + \sqrt{x})^3 dx$$

$$1.39.- \int \frac{dx}{\sqrt{5 - x^2}}$$

$$1.42.- \int \frac{dx}{x^2 + 5}$$

$$1.45.- \int \sqrt{x}(1 - \sqrt{x}) dx$$

$$1.48.- \int \frac{dx}{x^2 + 12}$$

$$1.51.- \int \frac{dx}{\sqrt{12 - x^2}}$$

$$1.54.- \int \frac{dx}{x\sqrt{12 + x^2}}$$

$$1.57.- \int \frac{dx}{\sqrt{2x^2 + 8}}$$

$$1.60.- \int \sqrt{10 - x^2} dx$$

$$1.63.- \int \sqrt{1 - \cos^2 x} dx$$

$$1.66.- \int \left(\tau gx - \frac{\operatorname{sen} x}{\cos x} \right) dx$$

$$1.69.- \int \sqrt{x^2 - \frac{3}{4}} dx$$

$$1.72.- \int \frac{dx}{x\sqrt{x^2 - 3}}$$

$$1.75.- \int \ell \eta |u| dx$$

$$1.78.- \int \frac{\sqrt{x} - \sqrt{2}}{\sqrt{2x}} dx$$

$$1.81.- \int \sqrt{x^2 + 11} dx$$

$$1.34.- \int (1 + \tau gx) dx$$

$$1.37.- \int (1 + \sqrt{x})^0 dx$$

$$1.40.- \int \frac{dx}{\sqrt{x^2 - 5}}$$

$$1.43.- \int \frac{dx}{x^2 - 5}$$

$$1.46.- \int (\tau g^2 x + 1) dx$$

$$1.49.- \int \frac{dx}{\sqrt{x^2 - 12}}$$

$$1.52.- \int \frac{dx}{x\sqrt{x^2 - 12}}$$

$$1.55.- \int \frac{dx}{\sqrt{8 - 2x^2}}$$

$$1.58.- \int \sqrt{x^2 - 10} dx$$

$$1.61.- \int \frac{1 - \cos^2 x}{\operatorname{sen}^2 x} dx$$

$$1.64.- \int (2^x - 3^x)^0 dx$$

$$1.67.- \int \frac{dx}{3^{-x}}$$

$$1.70.- \int \sqrt{x^2 + \frac{3}{4}} dx$$

$$1.73.- \int \frac{dx}{x\sqrt{x^2 + 3}}$$

$$1.76.- \int \exp(\ell \eta x) dx$$

$$1.79.- \int \sqrt{11 - x^2} dx$$

$$1.82.- \int \ell \eta (e^{\sqrt{x}}) dx$$

$$\begin{array}{lll}
1.83.- \int \left[\frac{1 + \sqrt{x} + \sqrt{x^3}}{1 - \sqrt{x}} \right] dx & 1.84.- \int (\tau g^2 x + \sec^2 x - 1) dx & 1.85.- \int \frac{dx}{\sqrt{3x^2 - 1}} \\
1.86.- \int (\operatorname{co} \tau g \theta - \operatorname{sen} \theta) dx & 1.87.- \int \frac{dx}{\sqrt{1 + 3x^2}} & 1.88.- \int \frac{dx}{\sqrt{1 - 3x^2}} \\
1.89.- \int \frac{dx}{1 + 3x^2} & 1.90.- \int \frac{dx}{3x^2 + 4} & 1.91.- \int \frac{dx}{3x^2 - 1} \\
1.92.- \int \frac{dx}{x\sqrt{3x^2 - 1}} & 1.93.- \int \frac{dx}{x\sqrt{1 + 3x^2}} & 1.94.- \int \frac{dx}{x\sqrt{1 - 3x^2}} \\
1.95.- \int \sqrt{1 - 3x^2} dx & 1.96.- \int \sqrt{1 + 3x^2} dx & 1.97.- \int \sqrt{3x^2 - 1} dx \\
1.98.- \int (3x^2 - 1) dx & 1.99.- \int (3x^2 - 1)^0 dx & 1.100.- \int (3x^2 - 1)^n du \\
1.101.- \int \exp(\ell \eta \frac{\sqrt{x}}{3}) dx & 1.102.- \int \ell \eta (e^{\frac{2x-1}{2}}) dx & 1.103.- \int (e^2 + e + 1)^x dx \\
1.104.- \int \left(\frac{1 + \tau g^2 x}{\sec^2 x} - 1 \right) dx & 1.105.- \int \exp(\ell \eta |1 + x|) dx & 1.106.- \int \sqrt{27 - x^2} dx \\
1.107.- \int \sqrt{x^2 - 27} dx & 1.108.- \int \sqrt{x^2 + 27} dx & 1.109.- \int \frac{dx}{3x\sqrt{x^2 - 1}} \\
1.110.- \int \frac{dx}{2x\sqrt{1 - x^2}} & 1.111.- \int \frac{dx}{5x\sqrt{x^2 + 1}} & 1.112.- \int \frac{dx}{3x\sqrt{9 - x^2}} \\
1.113.- \int \frac{dx}{4x\sqrt{x^2 + 16}} & 1.114.- \int \frac{dx}{5x\sqrt{x^2 - 25}} & 1.115.- \int \frac{(1 - \sqrt{x})^2}{x^2} dx \\
1.116.- \int (1 + \sqrt{x} + x)^2 dx & 1.117.- \int (1 - \sqrt{x} + x)^2 dx & 1.118.- \int (1 + x)^4 dx \\
1.119.- \int e^{\ell \eta \left| \frac{1 - \cos x}{2} \right|} dx & 1.120.- \int \exp \ell \eta \left(\frac{1 + x^2}{x^2} \right) dx & 1.121.- \int \ell \eta e^{\frac{1 - \operatorname{sen} x}{3}} dx \\
1.122.- \int (1 + \sqrt{x - 3x})^0 dx & 1.123.- \int \ell \eta e^{\frac{(1+x)^2}{2}} dx &
\end{array}$$

RESPUESTAS

$$1.32.- \int 3x^5 dx = 3 \int x^5 dx = \frac{3x^{5+1}}{5+1} + c = 3 \frac{x^6}{6} + c = \frac{x^6}{2} + c$$

$$1.33.- \int (1 + e)^x dx$$

$$\text{Sea: } a = 1 + e, \text{ Luego: } \int (1 + e)^x dx = \int a^x dx = \frac{a^x}{\ell \eta a} + c = \frac{(1 + e)^x}{\ell \eta (1 + e)} + c$$

$$1.34.- \int (1 + \tau g x) dx = \int dx + \int \tau g x dx = x + \ell \eta |\sec x| + c$$

$$1.35.- \int \cos^2 \frac{x}{2} dx = \int \frac{1 + \cos x}{2} dx = \frac{1}{2} \int dx + \frac{1}{2} \int \cos x dx = \frac{1}{2} x + \frac{1}{2} \operatorname{sen} x + c$$

$$1.36.- \int (1+\sqrt{x})^3 dx = \int (1+3\sqrt{x}+3(\sqrt{x^2})+\sqrt{x^3})dx = \int dx+3\sqrt{x}+3\int xdx + \int x^{\frac{3}{2}}dx$$

$$= x+2x^{\frac{3}{2}}+3\frac{x^2}{2}+\frac{2}{5}x^{\frac{5}{2}}+c = x+2x\sqrt{x}+3\frac{x^2}{2}+\frac{2}{5}x^2\sqrt{x}+c$$

$$1.37.- \int (1+\sqrt{x})^0 dx = \int dx = x+c$$

$$1.38.- \int \frac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}} dy = \frac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}} \int dy = \frac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}} y+c$$

$$1.39.- \int \frac{dx}{\sqrt{5-x^2}}$$

$$\text{Sea: } a = \sqrt{5}, \text{ Luego: } \int \frac{dx}{\sqrt{5-x^2}} = \int \frac{dx}{\sqrt{(\sqrt{5})^2-x^2}} = \arcsen \frac{x}{\sqrt{5}} + c = \arcsen \frac{\sqrt{5}x}{5} + c$$

$$1.40.- \int \frac{dx}{\sqrt{x^2-5}} = \int \frac{dx}{\sqrt{x^2-(\sqrt{5})^2}} = \ell \eta \left| x+\sqrt{x^2-5} \right| + c$$

$$1.41.- \int \frac{dx}{\sqrt{x^2+5}} = \int \frac{dx}{\sqrt{x^2+(\sqrt{5})^2}} = \ell \eta \left| x+\sqrt{x^2+5} \right| + c$$

$$1.42.- \int \frac{dx}{x^2+5}$$

$$\text{Sea: } a = \sqrt{5}, \text{ Luego: } \int \frac{dx}{x^2+(\sqrt{5})^2} = \frac{1}{\sqrt{5}} \arctan \frac{x}{\sqrt{5}} + c$$

$$= \frac{\sqrt{5}}{5} \arctan \frac{\sqrt{5}x}{5} + c$$

$$1.43.- \int \frac{dx}{x^2-5} = \int \frac{dx}{x^2-(\sqrt{5})^2} = \frac{1}{2\sqrt{5}} \ell \eta \left| \frac{x-\sqrt{5}}{x+\sqrt{5}} \right| + c = \frac{\sqrt{5}}{10} \ell \eta \left| \frac{x-\sqrt{5}}{x+\sqrt{5}} \right| + c$$

$$1.44.- \int (\operatorname{sen}^2 x + \cos^2 x - 1) dx = \int (1-1) dx = \int 0 dx = c$$

$$1.45.- \int \sqrt{x}(1-\sqrt{x}) dx = \int (\sqrt{x}-x) dx = \int \sqrt{x} dx - \int x dx = \frac{2}{3} x^{\frac{3}{2}} - \frac{x^2}{2} + c$$

$$1.46.- \int (\tau g^2 x + 1) dx = \int \sec^2 x dx = \tau g x + c$$

$$1.47.- \int \frac{dx}{x^2-12} = \int \frac{dx}{x^2-(\sqrt{12})^2} = \frac{1}{2\sqrt{12}} \ell \eta \left| \frac{x-\sqrt{12}}{x+\sqrt{12}} \right| + c = \frac{1}{4\sqrt{3}} \ell \eta \left| \frac{x-2\sqrt{3}}{x+2\sqrt{3}} \right| + c$$

$$= \frac{\sqrt{3}}{12} \ell \eta \left| \frac{x-2\sqrt{3}}{x+2\sqrt{3}} \right| + c$$

$$1.48.- \int \frac{dx}{x^2+12}$$

$$\text{Sea: } a = \sqrt{12}, \text{ Luego: } \int \frac{dx}{x^2+(\sqrt{12})^2} = \frac{1}{\sqrt{12}} \arctan \frac{x}{\sqrt{12}} + c$$

$$= \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{x}{2\sqrt{3}} + c = \frac{\sqrt{3}}{6} \operatorname{arctg} \frac{\sqrt{3}x}{6} + c$$

$$1.49.- \int \frac{dx}{\sqrt{x^2-12}} = \int \frac{dx}{\sqrt{x^2-(\sqrt{12})^2}} = \ell \eta \left| x + \sqrt{x^2-12} \right| + c$$

$$1.50.- \int \frac{dx}{\sqrt{x^2+12}} = \int \frac{dx}{\sqrt{x^2+(\sqrt{12})^2}} = \ell \eta \left| x + \sqrt{x^2+12} \right| + c$$

$$1.51.- \int \frac{dx}{\sqrt{12-x^2}}$$

Sea: $a = \sqrt{12}$,Luego: $\int \frac{dx}{\sqrt{12-x^2}} = \int \frac{dx}{\sqrt{(\sqrt{12})^2-x^2}}$

$$= \operatorname{arcsen} \frac{x}{\sqrt{12}} + c = \operatorname{arcsen} \frac{x}{2\sqrt{3}} + c = \operatorname{arcsen} \frac{\sqrt{3}x}{6} + c$$

$$1.52.- \int \frac{dx}{x\sqrt{x^2-12}} = \int \frac{dx}{x\sqrt{x^2-(\sqrt{12})^2}} = \frac{1}{\sqrt{12}} \operatorname{arcsec} \frac{x}{\sqrt{12}} + c = \frac{1}{2\sqrt{3}} \operatorname{arcsec} \frac{x}{2\sqrt{3}} + c$$

$$= \frac{\sqrt{3}}{6} \operatorname{arcsec} \frac{\sqrt{3}x}{6} + c$$

$$1.53.- \int \frac{dx}{x\sqrt{12-x^2}} = \int \frac{dx}{x\sqrt{(\sqrt{12})^2-x^2}} = \frac{1}{\sqrt{12}} \ell \eta \left| \frac{x}{\sqrt{12} + \sqrt{12-x^2}} \right| + c$$

$$= \frac{\sqrt{3}}{6} \ell \eta \left| \frac{x}{\sqrt{12} + \sqrt{12-x^2}} \right| + c$$

$$1.54.- \int \frac{dx}{x\sqrt{12+x^2}} = \frac{\sqrt{3}}{6} \ell \eta \left| \frac{x}{\sqrt{12} + \sqrt{12+x^2}} \right| + c$$

$$1.55.- \int \frac{dx}{\sqrt{8-2x^2}} = \int \frac{dx}{\sqrt{2(4-x^2)}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{4-x^2}} = \frac{1}{\sqrt{2}} \operatorname{arcsen} \frac{x}{2} + c = \frac{\sqrt{2}}{2} \operatorname{arcsen} \frac{x}{2} + c$$

$$1.56.- \int \frac{dx}{\sqrt{2x^2-8}} = \int \frac{dx}{\sqrt{2(x^2-4)}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{x^2-4}} = \frac{1}{\sqrt{2}} \ell \eta \left| x + \sqrt{x^2-4} \right| + c$$

$$= \frac{\sqrt{2}}{2} \ell \eta \left| x + \sqrt{x^2-4} \right| + c$$

$$1.57.- \int \frac{dx}{\sqrt{2x^2+8}} = \int \frac{dx}{\sqrt{2(x^2+4)}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{x^2+4}} = \frac{1}{\sqrt{2}} \ell \eta \left| x + \sqrt{x^2+4} \right| + c$$

$$= \frac{\sqrt{2}}{2} \ell \eta \left| x + \sqrt{x^2+4} \right| + c$$

$$1.58.- \int \sqrt{x^2-10} dx = \int \sqrt{x^2-(\sqrt{10})^2} dx = \frac{x}{2} \sqrt{x^2-10} - \frac{10}{2} \ell \eta \left| x + \sqrt{x^2-10} \right| + c$$

$$= \frac{x}{2} \sqrt{x^2 - 10} - 5 \ell \eta \left| x + \sqrt{x^2 - 10} \right| + c$$

$$1.59.- \int \sqrt{x^2 + 10} dx = \frac{x}{2} \sqrt{x^2 + 10} + 5 \ell \eta \left| x + \sqrt{x^2 + 10} \right| + c$$

$$1.60.- \int \sqrt{10 - x^2} dx = \int \sqrt{(\sqrt{10})^2 - x^2} dx = \frac{x}{2} \sqrt{10 - x^2} + \frac{10}{2} \arcsen \frac{x}{\sqrt{10}} + c$$

$$= \frac{x}{2} \sqrt{10 - x^2} + 5 \arcsen \frac{\sqrt{10}x}{10} + c$$

$$1.61.- \int \frac{1 - \cos^2 x}{\sen^2 x} dx = \int \frac{\sen^2 x}{\sen^2 x} dx = \int dx = x + c$$

$$1.62.- \int \sqrt{1 - \sen^2 x} dx = \int \sqrt{\cos^2 x} dx = \int \cos x dx = \sen x + c$$

$$1.63.- \int \sqrt{1 - \cos^2 x} dx = \int \sqrt{\sen^2 x} dx = \int \sen x dx = -\cos x + c$$

$$1.64.- \int (2^x - 3^x)^0 dx = \int dx = x + c$$

$$1.65.- \int (2^0 - 3^0)^n dx = \int (0)^n dx = \int 0 dx = c$$

$$1.66.- \int \left(\tau gx - \frac{\sen x}{\cos x} \right) dx = \int (\tau gx - \tau gx) dx = \int 0 dx = c$$

$$1.67.- \int \frac{dx}{3^{-x}} = \int 3^x dx = \frac{3^x}{\ell \eta 3} + c$$

$$1.68.- \int \sqrt{\frac{3}{4} - x^2} dx = \int \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 - x^2} dx = \frac{x}{2} \sqrt{\frac{3}{4} - x^2} + \frac{3/4}{2} \arcsen \frac{x}{\sqrt{3}/2} + c$$

$$= \frac{x}{2} \sqrt{\frac{3}{4} - x^2} + \frac{3}{8} \arcsen \frac{2x}{\sqrt{3}} + c$$

$$1.69.- \int \sqrt{x^2 - \frac{3}{4}} dx = \int \sqrt{x^2 - \left(\frac{\sqrt{3}}{2}\right)^2} dx = \frac{x}{2} \sqrt{x^2 - \frac{3}{4}} - \frac{3/4}{2} \ell \eta \left| x + \sqrt{x^2 - \frac{3}{4}} \right| + c$$

$$= \frac{x}{2} \sqrt{x^2 - \frac{3}{4}} - \frac{3}{8} \ell \eta \left| x + \sqrt{x^2 - \frac{3}{4}} \right| + c$$

$$1.70.- \int \sqrt{x^2 + \frac{3}{4}} dx = \int \sqrt{x^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx = \frac{x}{2} \sqrt{x^2 + \frac{3}{4}} + \frac{3}{8} \ell \eta \left| x + \sqrt{x^2 + \frac{3}{4}} \right| + c$$

$$1.71.- \int \frac{dx}{x\sqrt{3-x^2}} = \int \frac{dx}{x\sqrt{(\sqrt{3})^2 - x^2}} = \frac{1}{\sqrt{3}} \ell \eta \left| \frac{x}{\sqrt{3} + \sqrt{3-x^2}} \right| + c$$

$$= \frac{\sqrt{3}}{3} \ell \eta \left| \frac{x}{\sqrt{3} + \sqrt{3-x^2}} \right| + c$$

$$1.72.- \int \frac{dx}{x\sqrt{x^2-3}} = \frac{1}{\sqrt{3}} \arcsec \frac{x}{\sqrt{3}} + c = \frac{\sqrt{3}}{3} \arcsec \frac{\sqrt{3}x}{3} + c$$

$$1.73.- \int \frac{dx}{x\sqrt{x^2+3}} = \frac{\sqrt{3}}{3} \ell \eta \left| \frac{x}{\sqrt{3} + \sqrt{x^2+3}} \right| + c$$

$$1.74.- \int (\operatorname{sen}^{3x} \theta) dy = \operatorname{sen}^{3x} \theta \int dy = (\operatorname{sen}^{3x} \theta) y + c$$

$$1.75.- \int \ell \eta |u| dx = \ell \eta |u| \int dx = \ell \eta |u| x + c$$

$$1.76.- \int \exp(\ell \eta x) dx = \int x dx = \frac{x^2}{2} + c$$

$$1.77.- \int e^{\ell \eta x^2} dx = \int x^2 dx = \frac{x^3}{3} + c$$

$$1.78.- \int \frac{\sqrt{x} - \sqrt{2}}{\sqrt{2x}} dx = \int \frac{\sqrt{x}}{\sqrt{2x}} dx - \int \frac{\sqrt{2}}{\sqrt{2x}} dx = \int \sqrt{\frac{x}{2x}} dx - \int \sqrt{\frac{2}{2x}} dx = \frac{1}{\sqrt{2}} \int dx - \int \frac{1}{\sqrt{x}} dx = \\ = \frac{1}{\sqrt{2}} \int dx - \int x^{-\frac{1}{2}} dx = \frac{1}{\sqrt{2}} x - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{\sqrt{2}}{2} x - 2x^{\frac{1}{2}} + c$$

$$1.79.- \int \sqrt{11-x^2} dx = \frac{x}{2} \sqrt{11-x^2} + \frac{11}{2} \operatorname{arcsen} \frac{x}{\sqrt{11}} + c = \frac{x}{2} \sqrt{11-x^2} + \frac{11}{2} \operatorname{arcsen} \frac{\sqrt{11}x}{11} + c$$

$$1.80.- \int \sqrt{x^2-11} dx = \frac{x}{2} \sqrt{x^2-11} - \frac{11}{2} \ell \eta \left| x + \sqrt{x^2-11} \right| + c$$

$$1.81.- \int \sqrt{x^2+11} dx = \frac{x}{2} \sqrt{x^2+11} + \frac{11}{2} \ell \eta \left| x + \sqrt{x^2+11} \right| + c$$

$$1.82.- \int \ell \eta (e^{\sqrt{x}}) dx = \int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3} x \sqrt{x} + c$$

$$1.83.- \int \left[\frac{1 + \sqrt{x} + \sqrt{x^3}}{1 - \sqrt{x}} \right] dx = \int dx = x + c$$

$$1.84.- \int (\tau g^2 x + \sec^2 x - 1) dx = \int 0 dx = c$$

$$1.85.- \int \frac{dx}{\sqrt{3x^2-1}} = \int \frac{dx}{\sqrt{3} \sqrt{(x^2 - \frac{1}{3})}} = \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{(x^2 - \frac{1}{3})}} = \frac{1}{\sqrt{3}} \ell \eta \left| x + \sqrt{(x^2 - \frac{1}{3})} \right| + c \\ = \frac{\sqrt{3}}{3} \ell \eta \left| x + \sqrt{(x^2 - \frac{1}{3})} \right| + c$$

$$1.86.- \int (\operatorname{co} \tau g \theta - \operatorname{sen} \theta) dx = (\operatorname{co} \tau g \theta - \operatorname{sen} \theta) \int dx = (\operatorname{co} \tau g \theta - \operatorname{sen} \theta) x + c$$

$$1.87.- \int \frac{dx}{\sqrt{1+3x^2}} = \int \frac{dx}{\sqrt{3} \sqrt{\frac{1}{3} + x^2}} = \frac{\sqrt{3}}{3} \ell \eta \left| x + \sqrt{\frac{1}{3} + x^2} \right| + c$$

$$1.88.- \int \frac{dx}{\sqrt{1-3x^2}} = \int \frac{dx}{\sqrt{3} \sqrt{\frac{1}{3} - x^2}} = \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{\frac{1}{3} - x^2}} = \frac{1}{\sqrt{3}} \operatorname{arcsen} \frac{x}{\frac{1}{\sqrt{3}}} + c \\ = \frac{\sqrt{3}}{3} \operatorname{arcsen} \sqrt{3} x + c$$

$$1.89.- \int \frac{dx}{1+3x^2} = \int \frac{dx}{3(\frac{1}{3} + x^2)} = \frac{1}{3} \int \frac{dx}{\frac{1}{3} + x^2} = \frac{1}{3} \frac{1}{\frac{1}{\sqrt{3}}} \operatorname{arc} \tau g \frac{x}{\frac{1}{\sqrt{3}}} + c = \frac{\sqrt{3}}{3} \operatorname{arc} \tau g \sqrt{3} x + c$$

$$1.90.- \int \frac{dx}{3x^2+4} = \frac{1}{3} \int \frac{dx}{x^2+\frac{4}{3}} = \frac{1}{3} \frac{1}{\frac{2}{\sqrt{3}}} \operatorname{arctg} \frac{x}{\frac{2}{\sqrt{3}}} + c = \frac{\sqrt{3}}{6} \operatorname{arctg} \frac{\sqrt{3}x}{2} + c$$

$$1.91.- \int \frac{dx}{3x^2-1} = \frac{1}{3} \int \frac{dx}{x^2-\frac{1}{3}} = \frac{1}{3} \frac{1}{2\frac{1}{\sqrt{3}}} \ell \eta \left| \frac{x-\frac{1}{\sqrt{3}}}{x+\frac{1}{\sqrt{3}}} \right| + c = \frac{\sqrt{3}}{6} \ell \eta \left| \frac{\sqrt{3}x-1}{\sqrt{3}x+1} \right| + c$$

$$1.92.- \int \frac{dx}{x\sqrt{3x^2-1}} = \int \frac{dx}{\sqrt{3}x\sqrt{x^2-\frac{1}{3}}} = \frac{1}{\sqrt{3}} \int \frac{dx}{x\sqrt{x^2-\frac{1}{3}}} = \frac{1}{\sqrt{3}} \frac{1}{\frac{1}{\sqrt{3}}} \operatorname{arcsec} \frac{x}{\frac{1}{\sqrt{3}}} + c$$

$$= \operatorname{arcsec} \sqrt{3}x + c$$

$$1.93.- \int \frac{dx}{x\sqrt{1+3x^2}} = \frac{1}{\sqrt{3}} \int \frac{dx}{x\sqrt{\frac{1}{3}+x^2}} = \frac{1}{\sqrt{3}} \frac{1}{\frac{1}{\sqrt{3}}} \ell \eta \left| \frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}+x^2}} \right| + c$$

$$= \ell \eta \left| \frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}+x^2}} \right| + c$$

$$1.94.- \int \frac{dx}{x\sqrt{1-3x^2}} = \frac{1}{\sqrt{3}} \int \frac{dx}{x\sqrt{\frac{1}{3}-x^2}} = \ell \eta \left| \frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}-x^2}} \right| + c$$

$$1.95.- \int \sqrt{1-3x^2} dx = \sqrt{3} \int \sqrt{\frac{1}{3}-x^2} dx = \sqrt{3} \left[\frac{x}{2} \sqrt{\frac{1}{3}-x^2} + \frac{1}{2} \operatorname{arcsen} \frac{x}{\frac{1}{\sqrt{3}}} \right] + c$$

$$= \sqrt{3} \left[\frac{x}{2} \sqrt{\frac{1}{3}-x^2} + \frac{1}{6} \operatorname{arcsen} \sqrt{3}x \right] + c$$

$$1.96.- \int \sqrt{1+3x^2} dx = \sqrt{3} \int \sqrt{\frac{1}{3}+x^2} dx = \sqrt{3} \left[\frac{x}{2} \sqrt{\frac{1}{3}+x^2} + \frac{1}{2} \ell \eta \left| x + \sqrt{\frac{1}{3}+x^2} \right| \right] + c$$

$$= \sqrt{3} \left[\frac{x}{2} \sqrt{\frac{1}{3}+x^2} + \frac{1}{6} \ell \eta \left| x + \sqrt{\frac{1}{3}+x^2} \right| \right] + c$$

$$1.97.- \int \sqrt{3x^2-1} dx = \sqrt{3} \int \sqrt{x^2-\frac{1}{3}} dx = \sqrt{3} \left[\frac{x}{2} \sqrt{x^2-\frac{1}{3}} - \frac{1}{6} \ell \eta \left| x + \sqrt{x^2-\frac{1}{3}} \right| \right] + c$$

$$1.98.- \int (3x^2-1) dx = 3 \int x^2 dx - \int dx = x^3 - x + c$$

$$1.99.- \int (3x^2-1)^0 dx = \int dx = x + c$$

$$1.100.- \int (3x^2-1)^n du = (3x^2-1)^n \int du = (3x^2-1)^n u + c$$

$$1.101.- \int \exp(\ell \eta \frac{\sqrt{x}}{3}) dx = \int \frac{\sqrt{x}}{3} dx = \frac{1}{3} \int x^{\frac{1}{2}} dx = \frac{1}{3} \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{9} x^{\frac{3}{2}} + c$$

$$1.102.- \int \ell \eta (e^{\frac{2x-1}{2}}) dx = \int \frac{2x-1}{2} dx = \int x dx - \frac{1}{2} \int dx = \frac{x^2}{2} - \frac{1}{2} x + c$$

$$1.103.- \int (e^2 + e + 1)^x dx$$

Sea: $a = (e^2 + e + 1)$, Luego: $\int a^x dx = \frac{a^x}{\ell \eta a} + c = \frac{(e^2 + e + 1)^x}{\ell \eta (e^2 + e + 1)} + c$

1.104.- $\int \left(\frac{1 + \tau g^2 x}{\sec^2 x} - 1 \right) dx = \int (1 - 1) dx = \int 0 dx = c$

1.105.- $\int \exp(\ell \eta |1 + x|) dx = \int (1 + x) dx = \int dx + \int x dx = x + \frac{x^2}{2} + c$

1.106.- $\int \sqrt{27 - x^2} dx = \frac{x}{2} \sqrt{27 - x^2} + \frac{27}{2} \operatorname{arcsen} \frac{x}{3\sqrt{3}} + c$

1.107.- $\int \sqrt{x^2 - 27} dx = \frac{x}{2} \sqrt{x^2 - 27} - \frac{27}{2} \ell \eta \left| x + \sqrt{x^2 - 27} \right| + c$

1.108.- $\int \sqrt{x^2 + 27} dx = \frac{x}{2} \sqrt{x^2 + 27} + \frac{27}{2} \ell \eta \left| x + \sqrt{x^2 + 27} \right| + c$

1.109.- $\int \frac{dx}{3x\sqrt{x^2 - 1}} = \frac{1}{3} \int \frac{dx}{x\sqrt{x^2 - 1}} = \frac{1}{3} \operatorname{arc} \operatorname{sec} x + c$

1.110.- $\int \frac{dx}{2x\sqrt{1 - x^2}} = \frac{1}{2} \int \frac{dx}{x\sqrt{1 - x^2}} = \frac{1}{2} \ell \eta \left| \frac{x}{1 + \sqrt{1 - x^2}} \right| + c$

1.111.- $\int \frac{dx}{5x\sqrt{x^2 + 1}} = \frac{1}{5} \int \frac{dx}{x\sqrt{x^2 + 1}} = \frac{1}{5} \ell \eta \left| \frac{x}{1 + \sqrt{x^2 + 1}} \right| + c$

1.112.- $\int \frac{dx}{3x\sqrt{9 - x^2}} = \frac{1}{3} \int \frac{dx}{x\sqrt{9 - x^2}} = \frac{1}{3} \frac{1}{3} \ell \eta \left| \frac{x}{3 + \sqrt{9 - x^2}} \right| + c = \frac{1}{9} \ell \eta \left| \frac{x}{3 + \sqrt{9 - x^2}} \right| + c$

1.113.- $\int \frac{dx}{4x\sqrt{x^2 + 16}} = \frac{1}{4} \int \frac{dx}{x\sqrt{x^2 + 16}} = \frac{1}{4} \frac{1}{4} \ell \eta \left| \frac{x}{4 + \sqrt{x^2 + 16}} \right| + c$

$= \frac{1}{16} \ell \eta \left| \frac{x}{4 + \sqrt{x^2 + 16}} \right| + c$

1.114.- $\int \frac{dx}{5x\sqrt{x^2 - 25}} = \frac{1}{5} \int \frac{dx}{x\sqrt{x^2 - 25}} = \frac{1}{5} \frac{1}{5} \operatorname{arc} \operatorname{sec} \frac{x}{5} + c = \frac{1}{25} \operatorname{arc} \operatorname{sec} \frac{x}{5} + c$

1.115.- $\int \frac{(1 - \sqrt{x})^2}{x^2} dx = \int \frac{1 - 2\sqrt{x} + x}{x^2} dx = \int (x^{-2} - 2x^{-3/2} + x^{-1}) dx$

$= \int x^{-2} dx - \int 2x^{-3/2} dx + \int x^{-1} dx = -x^{-1} - 2 \frac{x^{-1/2}}{-1/2} + \ell \eta |x| + c = -x^{-1} - 2 \frac{x^{-1/2}}{-1/2} + \ell \eta |x| + c$

$= -x^{-1} + 4x^{-1/2} + \ell \eta |x| + c = -\frac{1}{x} + \frac{4}{\sqrt{x}} + \ell \eta |x| + c$

1.116.- $\int (1 + \sqrt{x} + x)^2 dx = (1 + x + x^2 + 2\sqrt{x} + 2x + 2x^{3/2}) dx$

$= \int (1 + 2x^{1/2} + 3x + 2x^{3/2} + x^2) dx = \int dx + 2 \int x^{1/2} dx + 3 \int x dx + 2 \int x^{3/2} dx + \int x^2 dx$

$x + \frac{2x^{3/2}}{3/2} + 3 \frac{x^2}{2} + 2 \frac{x^{5/2}}{5/2} + \frac{x^3}{3} + c = x + \frac{4x^{3/2}}{3} + 3 \frac{x^2}{2} + 4 \frac{x^{5/2}}{5} + \frac{x^3}{3} + c$

$$\begin{aligned}
 1.117.- \int (1 - \sqrt{x} + x)^2 dx &= \int (1 + x + x^2 - 2\sqrt{x} + 2x - 2x^{3/2}) dx \\
 &= \int (1 - 2x^{1/2} + 3x - 2x^{3/2} + x^2) dx = x - \frac{4x^{3/2}}{3} + 3\frac{x^2}{2} - 4\frac{x^{5/2}}{5} + \frac{x^3}{3} + c
 \end{aligned}$$

$$\begin{aligned}
 1.118.- \int (1+x)^4 dx &= \int (1+4x+6x^2+4x^3+x^4) dx \\
 &= \int dx + 4\int x dx + 6\int x^2 dx + 4\int x^3 dx + \int x^4 dx = x + 2x^2 + 2x^3 + x^4 + \frac{1}{5}x^5 + c
 \end{aligned}$$

$$1.119.- \int e^{\ell \eta \left| \frac{1-\cos x}{2} \right|} dx = \int \frac{1-\cos x}{2} dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos x dx = \frac{1}{2} x - \frac{1}{2} \sin x + c$$

$$1.120.- \int \exp \ell \eta \left(\frac{1+x^2}{x^2} \right) dx = \int \frac{1+x^2}{x^2} dx = \int \frac{1}{x^2} dx + \int dx = \int x^{-2} dx + \int dx = -\frac{1}{x} + x + c$$

$$1.121.- \int \ell \eta e^{\frac{1-\sin x}{3}} dx = \int \frac{1-\sin x}{3} dx = \frac{1}{3} \int dx - \frac{1}{3} \int \sin x dx = \frac{1}{3} x + \frac{1}{3} \cos x + c$$

$$1.122.- \int (1 + \sqrt{x-3x})^0 dx = \int dx = x + c$$

$$\begin{aligned}
 1.123.- \int \ell \eta e^{\frac{(1+x)^2}{2}} dx &= \int \frac{(1+x)^2}{2} dx = \int \frac{1+2x+x^2}{2} dx = \frac{1}{2} \int dx + \int x dx + \frac{1}{2} \int x^2 dx \\
 &= \frac{1}{2} x + \frac{x^2}{2} + \frac{x^3}{6} + c
 \end{aligned}$$