INTEGRACION DE FUNCIONES CUADRATICAS

Una función cuadrática, es de la forma: ax^2+bx+c y si ésta aparece en el denominador, la integral que la contiene se hace fácil de encontrar, para la cual conviene diferenciar dos tipos esenciales en lo que se refiere al numerador.

EJERCICIOS DESARROLLADOS

5.1.-Encontrar: $\int \frac{dx}{x^2+2x+5}$

Solución.- Completando cuadrados, se tiene:

$x^2+2x+5=(x^2+2x+\_\_)+5-\_\_=(x^2+2x+1)+5-1=(x^2+2x+1)+4$

$x^2+2x+5=(x+1)^2+2^2$, luego se tiene:

$\int \frac{dx}{x^2+2x+5}=\int \frac{dx}{(x+1)^2+2^2}$. Sea: $w=x+1, dw=dx; a=2$

$$\int \frac{dx}{(x+1)^2+2^2}=\int \frac{dw}{w^2+2^2}=\frac{1}{2}\operatorname{arc}\tau g\frac{w}{a}+c=\frac{1}{2}\operatorname{arc}\tau g\frac{x+1}{2}+c$$

Respuesta: $\int \frac{dx}{x^2+2x+5}=\frac{1}{2}\operatorname{arc}\tau g\frac{x+1}{2}+c$

5.2.-Encontrar: $\int \frac{dx}{4x^2+4x+2}$

Solución.- $\int \frac{dx}{4x^2+4x+2}=\int \frac{dx}{4(x^2+x+1/2)}=\frac{1}{4}\int \frac{dx}{x^2+x+1/2}$

Completando cuadrados:

$$x^2+x+1/2=(x^2+x+\_\_)+\frac{1}{2}-\_\_=(x^2+x+\frac{1}{4})+\frac{1}{2}-\frac{1}{4}=(x^2+x+\frac{1}{4})+\frac{1}{4}$$

$(x^2+x+\frac{1}{2})=(x+\frac{1}{2})^2+(\frac{1}{2})^2$, luego se tiene:

$\frac{1}{4}\int \frac{dx}{x^2+x+1/2}=\frac{1}{4}\int \frac{dx}{(x+1/2)^2+(1/2)^2}$, Sea: $w=x+1/2, dw=dx; a=1/2$

$$=\frac{1}{4}\int \frac{dx}{(x+1/2)^2+(1/2)^2}=\frac{1}{4}\int \frac{dw}{w^2+a^2}=\frac{1}{4}\frac{1}{a}\operatorname{arc}\tau g\frac{w}{a}+c=\frac{1}{4}\frac{1}{1/2}\operatorname{arc}\tau g\frac{x+1/2}{1/2}+c$$

$$=\frac{1}{2}\operatorname{arc}\tau g\frac{\frac{2x+1}{\cancel{2}}}{\frac{1}{\cancel{2}}}+c=\frac{1}{2}\operatorname{arc}\tau g(2x+1)+c$$

Respuesta: $\int \frac{dx}{4x^2+4x+2} = \frac{1}{2}\operatorname{arc}\tau g(2x+1)+c$

5.3.-Encontrar: $\int \frac{2xdx}{x^2-x+1}$

Solución.- $u = x^2 - x + 1, du = (2x-1)dx$

$$\int \frac{2xdx}{x^2-x+1} = \int \frac{(2x-1+1)dx}{x^2-x+1} = \int \frac{(2x-1)dx}{x^2-x+1} + \int \frac{dx}{x^2-x+1} = \int \frac{du}{u} + \int \frac{dx}{x^2-x+1}$$

Completando cuadrados:

$$x^2-x+1 = (x^2-x+\_\_)+1\_\_ = (x^2-x+\frac{1}{4})+1-\frac{1}{4}$$

$x^2-x+1 = (x^2-\frac{1}{2})^2+\frac{3}{4}$, Luego se tiene:

$$\int \frac{du}{u} + \int \frac{dx}{x^2-x+1} = \int \frac{du}{u} + \int \frac{du}{(x-\frac{1}{2})^2+\frac{3}{4}} = \int \frac{du}{u} + \int \frac{dx}{(x-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

$w = x - \frac{1}{2}, dw = dx; a = \frac{\sqrt{3}}{2}$, luego:

$$\int \frac{du}{u} + \int \frac{dx}{(x-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} = \int \frac{du}{u} + \int \frac{dw}{w^2+a^2} = \ell\eta|u| + \frac{1}{a}\operatorname{arc}\tau g\frac{w}{a}+c$$

$$= \ell\eta\left|x^2-x+1\right| + \frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arc}\tau g\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}+c = \ell\eta\left|x^2-x+1\right| + \frac{2\sqrt{3}}{3}\operatorname{arc}\tau g\frac{\frac{2x-1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}}+c$$

Respuesta: $\int \frac{2xdx}{x^2-x+1} = \ell\eta\left|x^2-x+1\right| + \frac{2\sqrt{3}}{3}\operatorname{arc}\tau g\frac{2x-1}{\sqrt{3}}+c$

5.4.-Encontrar: $\int \frac{x^2dx}{x^2+2x+5}$

Solución.-

$$\int \frac{x^2dx}{x^2+2x+5} = \int\left(1-\frac{2x+5}{x^2+2x+5}\right)dx = \int dx - \int \frac{2x+5}{x^2+2x+5}dx,$$

Sea: $u = x^2+2x+5, du = (2x+2)dx$

Ya se habrá dado cuenta el lector que tiene que construir en el numerador, la expresión: $(2x+2)dx$. Luego se tiene:

$$= \int dx - \int \frac{(2x+2+3)}{x^2+2x+5}dx = \int dx - \int \frac{(2x+2)dx}{x^2+2x+5} + 3\int \frac{dx}{x^2+2x+5},$$

Completando cuadrados, se tiene:

$$x^2+2x+5 = (x^2+2x+\_\_)+5-\_\_ = (x^2+2x+1)+5-1 = (x^2+2x+1)+4 = (x+1)^2+2^2$$

Luego se admite como forma equivalente a la anterior:

$\int dx - \int \frac{du}{u} - 3\int \frac{dx}{(x+1)^2+2^2}$, Sea: $w = x+1, dw = dx; a = 2$, luego:

$$= \int dx - \int \frac{du}{u} - 3\int \frac{dw}{w^2 + a^2} = x - \ell\eta\left|u\right| - 3\frac{1}{a}\operatorname{arc}\tau g\,\frac{w}{a} + c$$

$$= x - \ell\eta\left|x^2 + 2x + 5\right| - \frac{3}{2}\operatorname{arc}\tau g\,\frac{x+1}{2} + c$$

Respuesta: $\int \frac{x^2dx}{x^2 + 2x + 5} = x - \ell\eta\left|x^2 + 2x + 5\right| - \frac{3}{2}\operatorname{arc}\tau g\,\frac{x+1}{2} + c$

5.5.-Encontrar: $\int \frac{2x - 3}{x^2 + 2x + 2}dx$

Solución.- Sea: $u = x^2 + 2x + 2, du = (2x + 2)dx$

$$\int \frac{2x - 3}{x^2 + 2x + 2}dx = \int \frac{2x + 2 - 5}{x^2 + 2x + 2}dx = \int \frac{2x + 2}{x^2 + 2x + 2}dx - 5\int \frac{dx}{x^2 + 2x + 2}$$

$= \int \frac{du}{u}dx - 5\int \frac{dx}{x^2 + 2x + 2}$, Completando cuadrados:

$x^2 + 2x + 2 = (x + 1)^2 + 1^2$. Luego:

$= \int \frac{du}{u}dx - 5\int \frac{dx}{(x + 1)^2 + 1^2}$, Sea: $w = x + 1, du = dx; a = 1$. Entonces se tiene:

$$= \int \frac{du}{u}dx - 5\int \frac{dx}{w^2 + a^2} = \ell\eta\left|u\right| - 5\frac{1}{a}\operatorname{arc}\tau g\,\frac{w}{a} + c = \ell\eta\left|x^2 + 2x + 5\right| - 5\operatorname{arc}\tau g(x + 1) + c$$

Respuesta: $\int \frac{2x - 3}{x^2 + 2x + 2}dx = \ell\eta\left|x^2 + 2x + 5\right| - 5\operatorname{arc}\tau g(x + 1) + c$

5.6.-Encontrar: $\int \frac{dx}{\sqrt{x^2 - 2x - 8}}$

Solución.- Completando cuadrados se tiene: $x^2 - 2x - 8 = (x - 1)^2 - 3^2$

$\int \frac{dx}{\sqrt{x^2 - 2x - 8}} = \int \frac{dx}{\sqrt{(x - 1)^2 - 3^2}}$, Sea: $w = x - 1, dw = dx; a = 3$

$$= \int \frac{dw}{\sqrt{w^2 - a^2}} = \ell\eta\left|w + \sqrt{w^2 - a^2}\right| + c = \ell\eta\left|x - 1 + \sqrt{x^2 - 2x - 8}\right| + c$$

Respuesta: $\int \frac{dx}{\sqrt{x^2 - 2x - 8}} = \ell\eta\left|x - 1 + \sqrt{x^2 - 2x - 8}\right| + c$

5.7.-Encontrar: $\int \frac{xdx}{\sqrt{x^2 - 2x + 5}}$

Solución.- Sea: $u = x^2 - 2x + 5, du = (2x - 2)dx$. Luego:

$$\int \frac{xdx}{\sqrt{x^2 - 2x + 5}} = \frac{1}{2}\int \frac{2xdx}{\sqrt{x^2 - 2x + 5}} = \frac{1}{2}\int \frac{2x - 2 + 2}{\sqrt{x^2 - 2x + 5}}dx$$

$$= \frac{1}{2}\int \frac{(2x - 2)dx}{\sqrt{x^2 - 2x + 5}} + \frac{2}{2}\int \frac{dx}{\sqrt{x^2 - 2x + 5}} = \frac{1}{2}\int \frac{du}{\sqrt{u}} + \int \frac{dx}{\sqrt{x^2 - 2x + 5}}$$

Completando cuadrados se tiene: $x^2 + 2x + 5 = (x - 1)^2 + 2^2$. Por lo tanto:

$= \frac{1}{2}\int u^{-1/2}du + \int \frac{dx}{\sqrt{(x-1)^2+2^2}}$. Sea: $w = x-1, du = dx; a = 2$

$= \frac{1}{2}\int u^{-1/2}du + \int \frac{dw}{\sqrt{w^2+a^2}} = \frac{1}{\cancel{2}}\frac{u^{1/2}}{1/\cancel{2}} + \ell\eta\left|w+\sqrt{w^2+a^2}\right| + c = u^{1/2} + \ell\eta\left|w+\sqrt{w^2+a^2}\right| + c$

$= \sqrt{x^2+2x+5} + \ell\eta\left|x-1+\sqrt{x^2-2x+5}\right| + c$

Respuesta: $\int \frac{xdx}{\sqrt{x^2-2x+5}} = \sqrt{x^2-2x+5} + \ell\eta\left|x-1+\sqrt{x^2-2x+5}\right| + c$

5.8.-Encontrar: $\int \frac{(x+1)dx}{\sqrt{2x-x^2}}$

Solución.- Sea: $u = 2x - x^2, du = (2-2x)dx$ .Luego:

$\int \frac{(x+1)dx}{\sqrt{2x-x^2}} = -\frac{1}{2}\int \frac{-2(x+1)dx}{\sqrt{2x-x^2}} = -\frac{1}{2}\int \frac{(-2x-2)dx}{\sqrt{2x-x^2}} = -\frac{1}{2}\int \frac{(-2x+2-4)dx}{\sqrt{2x-x^2}}$

$= -\frac{1}{2}\int \frac{(2-2x)dx}{\sqrt{2x-x^2}} + \frac{4}{2}\int \frac{dx}{\sqrt{2x-x^2}} = -\frac{1}{2}\int \frac{du}{\sqrt{u}} + 2\int \frac{dx}{\sqrt{2x-x^2}}$

Completando cuadrados: $2x - x^2 = -(x^2-2x) = -(x^2-2x+1-1) = -(x^2-2x+1)+1$

$= -(x-1)^2 + 1 = 1-(x-1)^2$. Luego la expresión anterior es equivalente a:

$= -\frac{1}{2}\int u^{-1/2}du + 2\int \frac{dx}{\sqrt{1-(x-1)^2}}$. Sea: $w = x-1, dw = dx; a = 1$. Entonces:

$= -\frac{1}{\cancel{2}}\int \frac{u^{1/2}}{1/\cancel{2}}du + 2\int \frac{dw}{\sqrt{a^2-w^2}} = -u^{1/2} + 2\,\mathrm{arcs}e\mathrm{n}\frac{w}{a} + c = -\sqrt{2x-x^2} + 2\,\mathrm{arcs}e\mathrm{n}(x-1) + c$

Respuesta: $\int \frac{(x+1)dx}{\sqrt{2x-x^2}} = -\sqrt{2x-x^2} + 2\,\mathrm{arcs}e\mathrm{n}(x-1) + c$

5.9.-Encontrar: $\int \frac{xdx}{\sqrt{5x^2-2x+1}}$

Solución.- Sea: $u = 5x^2-2x+1, du = (10x-2)dx$. Luego:

$\int \frac{xdx}{\sqrt{5x^2-2x+1}} = \frac{1}{10}\int \frac{10xdx}{\sqrt{5x^2-2x+1}} = \frac{1}{10}\int \frac{(10x-2+2)dx}{\sqrt{5x^2-2x+1}}$

$= \frac{1}{10}\int \frac{(10x-2)dx}{\sqrt{5x^2-2x+1}} + \frac{2}{10}\int \frac{dx}{\sqrt{5x^2-2x+1}} = \frac{1}{10}\int \frac{du}{\sqrt{u}} + \frac{1}{5}\int \frac{dx}{\sqrt{5x^2-2x+1}}$

$= \frac{1}{10}\int \frac{du}{\sqrt{u}} + \frac{1}{5}\int \frac{dx}{\sqrt{5(x^2-2/5\,x+1/5)}} = \frac{1}{10}\int u^{-1/2}du + \frac{1}{5\sqrt{5}}\int \frac{dx}{\sqrt{(x^2-2/5\,x+1/5)}}$

Completando cuadrados: $x^2 - \frac{2}{5}x + \frac{1}{5} = (x^2 - \frac{2}{5}x + \_\_) + \frac{1}{5} - \_\_$

$= (x^2 - \frac{2}{5}x + \frac{1}{25}) + \frac{1}{5} - \frac{1}{25} = (x - 1/5)^2 + (2/5)^2$, Luego es equivalente:

$= \frac{1}{10}\int u^{-1/2}du + \frac{1}{5\sqrt{5}}\int \frac{dx}{\sqrt{(x-1/5)^2+(2/5)^2}}$, Sea: $w = x - 1/5, dw = dx; a = 2/5$,

Entonces: $= \frac{1}{10}\int u^{-1/2}du + \frac{1}{5\sqrt{5}}\int \frac{dw}{\sqrt{w^2+a^2}} = \frac{1}{10}\frac{u^{1/2}}{1/2} + \frac{1}{5\sqrt{5}}\ell\eta\left|w+\sqrt{w^2+a^2}\right| + c$

$= \frac{\sqrt{5x^2-2x+1}}{5} + \frac{1}{5\sqrt{5}}\ell\eta\left|x - \frac{1}{5} + \frac{\sqrt{5x^2-2x+1}}{\sqrt{5}}\right| + c$

Respuesta: $\int \frac{xdx}{\sqrt{5x^2-2x+1}} = \frac{\sqrt{5x^2-2x+1}}{5} + \frac{\sqrt{5}}{25}\ell\eta\left|x - \frac{1}{5} + \frac{\sqrt{5x^2-2x+1}}{\sqrt{5}}\right| + c$

5.10.-Encontrar: $\int \frac{xdx}{\sqrt{5+4x-x^2}}$

Solución.- $u = 5+4x-x^2, du = (4-2x)dx$. Luego:

$\int \frac{xdx}{\sqrt{5+4x-x^2}} = -\frac{1}{2}\int \frac{-2xdx}{\sqrt{5+4x-x^2}} = -\frac{1}{2}\int \frac{(-2x+4-4)dx}{\sqrt{5+4x-x^2}}$

$= -\frac{1}{2}\int \frac{(4-2x)dx}{\sqrt{5+4x-x^2}} + \frac{4}{2}\int \frac{dx}{\sqrt{5+4x-x^2}} = -\frac{1}{2}\int \frac{du}{\sqrt{u}} + 2\int \frac{dx}{\sqrt{5+4x-x^2}}$

Completando cuadrados: $5+4x-x^2 = -(x^2-4x-5) = -(x^2-4x+4-4-5)$

$= -(x^2-4x+4)+9 = 9-(x-2)^2 = 3^2-(x-2)^2$. Equivalente a:

$= -\frac{1}{2}\int u^{-1/2}du + 2\int \frac{dx}{\sqrt{3^2-(x-2)^2}}$. Sea: $w = x-2, dw = dx; a = 3$. Entonces:

$= -\frac{1}{2}\int u^{-1/2}du + 2\int \frac{dw}{\sqrt{a^2-w^2}} = -\frac{1}{\cancel{2}}\frac{u^{1/2}}{1/\cancel{2}} + 2\operatorname{arcsen}\frac{w}{a} + c$

$= -\sqrt{5+4x-x^2} + 2\operatorname{arcsen}\frac{x-2}{3} + c$

Respuesta: $\int \frac{xdx}{\sqrt{5+4x-x^2}} = -\sqrt{5+4x-x^2} + 2\operatorname{arcsen}\frac{x-2}{3} + c$

5.11.-Encontrar: $\int \frac{dx}{\sqrt{2+3x-2x^2}}$

Solución.- Completando cuadrados se tiene:

$2+3x-2x^2 = -(2x^2-3x-2) = -2(x^2-3/2\,x-1) = -2(x^2-\frac{3}{2}x+\frac{9}{16}-\frac{25}{16})$

$= -2\left[(x^2-\frac{3}{2}x+\frac{9}{16})-\frac{25}{16}\right] = -2\left[(x-3/4)^2-(5/4)^2\right] = 2\left[(5/4)^2-(x-3/4)^2\right]$, luego:

$\int \frac{dx}{\sqrt{2+3x-2x^2}} = \int \frac{dx}{\sqrt{2\left[(5/4)^2-(x-3/4)^2\right]}} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{(5/4)^2-(x-3/4)^2}}$

Sea: $w = x-3/4, dw = dx, a = 5/4$. Luego:

$$=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{(5/4)^2-(x-3/4)^2}}=\frac{1}{\sqrt{2}}\int\frac{dw}{\sqrt{a^2-w^2}}=\frac{1}{\sqrt{2}}\operatorname{arcsen}\frac{w}{a}+c=\frac{1}{\sqrt{2}}\operatorname{arcsen}\frac{x-3/4}{5/4}+c$$

$$=\frac{\sqrt{2}}{2}\operatorname{arcsen}\frac{4x-3}{5}+c$$

Respuesta: $\int\frac{dx}{\sqrt{2+3x-2x^2}}=\frac{\sqrt{2}}{2}\operatorname{arcsen}\frac{4x-3}{5}+c$

5.12.-Encontrar: $\int\frac{dx}{3x^2+12x+42}$

Solución.-

$$\int\frac{dx}{3x^2+12x+42}=\int\frac{dx}{3(x^2+4x+14)}=\frac{1}{3}\int\frac{dx}{(x^2+4x+14)}=\frac{1}{3}\int\frac{dx}{(x^2+4x+4+10)}=$$

$$=\frac{1}{3}\int\frac{dx}{(x+2)^2+10}=\frac{1}{3}\int\frac{dx}{(x+2)^2+(\sqrt{10})^2}=\frac{1}{3}\frac{1}{\sqrt{10}}\operatorname{arc}\tau g\frac{x+2}{\sqrt{10}}+c$$

Respuesta: $\int\frac{dx}{3x^2+12x+42}=\frac{\sqrt{10}}{30}\operatorname{arc}\tau g\frac{x+2}{\sqrt{10}}+c$

5.13.-Encontrar: $\int\frac{3x-2}{x^2-4x+5}dx$

Solución.- Sea: $u=x^2-4x+5, du=(2x-4)dx$, Luego:

$$\int\frac{3x-2}{x^2-4x+5}dx=3\int\frac{xdx}{x^2-4x+5}-2\int\frac{dx}{x^2-4x+5}=3\int\frac{(x-2)+2}{x^2-4x+5}-2\int\frac{dx}{x^2-4x+5}$$

$$=3\int\frac{(x-2)}{x^2-4x+5}+6\int\frac{dx}{x^2-4x+5}-2\int\frac{dx}{x^2-4x+5}=\frac{3}{2}\int\frac{du}{u}+4\int\frac{dx}{x^2-4x+5}$$

$$=\frac{3}{2}\int\frac{du}{u}+4\int\frac{dx}{(x^2-4x+4)+1}=\frac{3}{2}\ell\eta\left|x^2-4x+5\right|+4\int\frac{dx}{(x-2)^2+1}$$

$$=\frac{3}{2}\ell\eta\left|x^2-4x+5\right|+4\operatorname{arc}\tau g(x-2)+c$$

Respuesta: $\int\frac{3x-2}{x^2-4x+5}dx=\frac{3}{2}\ell\eta\left|x^2-4x+5\right|+4\operatorname{arc}\tau g(x-2)+c$

EJERCICIOS PROPUESTOS

Usando Esencialmente la técnica tratada, encontrar las integrales siguientes:

5.14.- $\int\sqrt{x^2+2x-3}\,dx$ **5.15.-** $\int\sqrt{12+4x-x^2}\,dx$ **5.16.-** $\int\sqrt{x^2+4x}\,dx$

5.17.- $\int\sqrt{x^2-8x}\,dx$ **5.18.-** $\int\sqrt{6x-x^2}\,dx$ **5.19.-** $\int\frac{(5-4x)dx}{\sqrt{12x-4x^2-8}}$

5.20.- $\int \frac{xdx}{\sqrt{27+6x-x^2}}$

5.21.- $\int \frac{(x-1)dx}{3x^2-4x+3}$

5.22.- $\int \frac{(2x-3)dx}{x^2+6x+15}$

5.23.- $\int \frac{dx}{4x^2+4x+10}$

5.24.- $\int \frac{(2x+2)dx}{x^2-4x+9}$

5.25.- $\int \frac{(2x+4)dx}{\sqrt{4x-x^2}}$

5.26.- $\frac{2}{3}\int \frac{(x+\frac{3}{2})dx}{9x^2-12x+8}$

5.27.- $\int \frac{(x+6)dx}{\sqrt{5-4x-x^2}}$

5.28.- $\int \frac{dx}{2x^2+20x+60}$

5.29.- $\int \frac{3dx}{\sqrt{80+32x-4x^2}}$

5.30.- $\int \frac{dx}{\sqrt{12x-4x^2-8}}$

5.31.- $\int \frac{5dx}{\sqrt{28-12x-x^2}}$

5.32.- $\int \sqrt{12-8x-4x^2}dx$

5.33.- $\sqrt{x^2-x+\frac{5}{4}}dx$

5.34.- $\int \frac{dx}{x^2-2x+5}$

5.35.- $\int \frac{(1-x)dx}{\sqrt{8+2x-x^2}}$

5.36.- $\int \frac{xdx}{x^2+4x+5}$

5.37.- $\int \frac{(2x+3)dx}{4x^2+4x+5}$

5.38.- $\int \frac{(x+2)dx}{x^2+2x+2}$

5.39.- $\int \frac{(2x+1)dx}{x^2+8x-2}$

5.40.- $\int \frac{dx}{\sqrt{-x^2-6x}}$

5.41.- $\int \frac{(x-1)dx}{x^2+2x+2}$

RESPUESTAS

5.14.- $\int \sqrt{x^2-2x-3}dx$

Solución.- Completando cuadrados se tiene:

$x^2-2x-3=(x^2-2x+1)-3-1=(x-1)^2-4=(x-1)^2-2^2$

Haciendo: $u=x-1, du=dx; a=2$, se tiene:

$$\int \sqrt{x^2-2x-3}dx=\int \sqrt{(x-1)^2-2^2}dx=\int \sqrt{u^2-a^2}du$$

$$=\frac{1}{2}u\sqrt{u^2-a^2}-\frac{1}{2}a^2\ell\eta\left|u+\sqrt{u^2-a^2}\right|+c$$

$$=\frac{1}{2}(x-1)\sqrt{(x-1)^2-2^2}-\frac{1}{2}2^2\ell\eta\left|(x-1)+\sqrt{(x-1)^2-2^2}\right|+c$$

$$=\frac{1}{2}(x-1)\sqrt{x^2-2x-3}-2\ell\eta\left|(x-1)+\sqrt{x^2-2x-3}\right|+c$$

5.15.- $\int \sqrt{12+4x-x^2}dx$

Solución.- Completando cuadrados se tiene:

$12+4x-x^2=-(x^2-4x-12)=-(x^2-4x+4-12-4)=-(x^2-4x+4)+16$

$=4^2-(x-2)^2$

Haciendo: $u=x-2, du=dx; a=4$, se tiene:

$$\int \sqrt{12+4x-x^2}dx=\int \sqrt{4^2-(x-2)^2}dx=\int \sqrt{a^2-u^2}du=\frac{1}{2}u\sqrt{a^2-u^2}+\frac{1}{2}a^2\operatorname{arcsen}\frac{u}{a}+c$$

$$=\frac{1}{2}(x-2)\sqrt{4^2-(x-2)^2}+\frac{1}{2}4^2\,\text{arcsen}\frac{(x-2)}{4}+c$$

$$=\frac{1}{2}(x-2)\sqrt{12+4x-x^2}+8\,\text{arcsen}\frac{(x-2)}{4}+c$$

5.16.- $\int\sqrt{x^2+4x}\,dx$

Solución.- Completando cuadrados se tiene:

$x^2+4x=(x^2+4x+4)-4=(x+2)^2-2^2$

Haciendo: $u=x+2, du=dx; a=2$, se tiene:

$$\int\sqrt{x^2+4x}\,dx=\int\sqrt{(x+2)^2-2^2}\,dx=\int\sqrt{u^2-a^2}\,du$$

$$=\frac{1}{2}u\sqrt{u^2-a^2}-\frac{1}{2}a^2\ell\eta\left|u+\sqrt{u^2-a^2}\right|+c$$

$$=\frac{1}{2}(x+2)\sqrt{(x+2)^2-2^2}-\frac{1}{2}2^2\ell\eta\left|(x+2)+\sqrt{(x+2)^2-2^2}\right|+c$$

$$=\frac{(x+2)}{2}\sqrt{x^2+4x}-2\ell\eta\left|(x+2)+\sqrt{x^2+4x}\right|+c$$

5.17.- $\int\sqrt{x^2-8x}\,dx$

Solución.- Completando cuadrados se tiene:

$x^2-8x=(x^2-8x+16)-16=(x-4)^2-4^2$

Haciendo: $u=x-4, du=dx; a=4$, se tiene:

$$\int\sqrt{(x-4)^2-4^2}\,dx=\sqrt{u^2-a^2}\,du=\frac{1}{2}u\sqrt{u^2-a^2}-\frac{1}{2}a^2\ell\eta\left|u+\sqrt{u^2-a^2}\right|+c$$

$$=\frac{1}{2}(x-4)\sqrt{(x-4)^2-4^2}-\frac{1}{2}4^2\ell\eta\left|(x-4)+\sqrt{(x-4)^2-4^2}\right|+c$$

$$=\frac{(x-4)}{2}\sqrt{x^2-8x}-8\ell\eta\left|(x-4)+\sqrt{x^2-8x}\right|+c$$

5.18.- $\int\sqrt{6x-x^2}\,dx$

Solución.- Completando cuadrados se tiene:

$6x-x^2=-(x^2-6x)=-(x^2-6x+9-9)=-(x^2-6x+9)+9=3^2-(x-3)^2$

Haciendo: $u=x-3, du=dx; a=3$, se tiene:

$$\int\sqrt{6x-x^2}\,dx=\sqrt{3^2-(x-3)^2}\,dx=\sqrt{a^2-u^2}\,du=\frac{1}{2}u\sqrt{a^2-u^2}+\frac{1}{2}a^2\,\text{arcsen}\frac{u}{a}+c$$

$$=\frac{1}{2}(x-3)\sqrt{3^2-(x-3)^2}+\frac{1}{2}3^2\,\text{arcsen}\frac{x-3}{3}+c$$

$$=\frac{(x-3)}{2}\sqrt{6x-x^2}+\frac{9}{2}\text{arcsen}\frac{x-3}{3}+c$$

5.19.- $\int\frac{(5-4x)dx}{\sqrt{12x-4x^2-8}}$

Solución.- Sea: $u=12x-4x^2-8, du=(12-8x)dx$

$$\int\frac{(5-4x)dx}{\sqrt{12x-4x^2-8}}=\int\frac{(-4x+5)dx}{\sqrt{12x-4x^2-8}}=\frac{1}{2}\int\frac{2(-4x+5)dx}{\sqrt{12x-4x^2-8}}=\frac{1}{2}\int\frac{(-8x+10)dx}{\sqrt{12x-4x^2-8}}$$

$$=\frac{1}{2}\int\frac{(-8x+12-2)dx}{\sqrt{12x-4x^2-8}}=\frac{1}{2}\int\frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}}-\int\frac{dx}{\sqrt{12x-4x^2-8}}$$

$$=\frac{1}{2}\int\frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}}-\int\frac{dx}{\sqrt{4(3x-x^2-2)}}=\frac{1}{2}\int\frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}}-\frac{1}{2}\int\frac{dx}{\sqrt{3x-x^2-2}}$$

Completando cuadrados se tiene:

$$3x-x^2-2=-(x^2-3x+2)=-(x^2-3x+\frac{9}{4}-\frac{9}{4}+2)=-(x^2-3x+\frac{9}{4})+\frac{9}{4}-2$$

$$=-(x-3/2)^2+\frac{1}{4}=(\frac{1}{2})^2-(x-\frac{3}{2})^2$$

$$=\frac{1}{2}\int\frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}}-\frac{1}{2}\int\frac{dx}{\sqrt{(1/2)^2-(x-3/2)^2}}$$

Haciendo: $u=12x-4x^2-8, du=(12-8x)dx$ y $w=x-3/2, dw=dx$, entonces:

$$=\frac{1}{2}\int\frac{du}{\sqrt{u}}-\frac{1}{2}\int\frac{dw}{\sqrt{(1/2)^2-w^2}}=\frac{1}{\cancel{2}}\frac{u^{1/2}}{1/\cancel{2}}-\frac{1}{2}\operatorname{arcsen}\frac{w}{1/2}+c$$

$$=u^{1/2}-\frac{1}{2}\operatorname{arcsen}2w+c=\sqrt{12x-4x^2-8}-\frac{1}{2}\operatorname{arcsen}(2x-3)+c$$

5.20.- $\int\frac{xdx}{\sqrt{27+6x-x^2}}$

Solución.- Sea: $u=27+6x-x^2, du=(6-2x)dx$

$$\int\frac{xdx}{\sqrt{27+6x-x^2}}=-\frac{1}{2}\int\frac{-2xdx}{\sqrt{27+6x-x^2}}=-\frac{1}{2}\int\frac{(-2x+6-6)dx}{\sqrt{27+6x-x^2}}$$

$$=-\frac{1}{2}\int\frac{(-2x+6)dx}{\sqrt{27+6x-x^2}}+3\int\frac{dx}{\sqrt{27+6x-x^2}}=-\frac{1}{2}\int\frac{du}{\sqrt{u}}+3\int\frac{dx}{\sqrt{27+6x-x^2}}$$

Completando cuadrados se tiene:

$$27+6x-x^2=-(x^2-6x-27)=-(x^2-6x+9-9-27)=-(x^2-6x+9)+36$$

$=6^2-(x-3)^2$, Luego:

$$=-\frac{1}{2}\int u^{-1/2}du+3\int\frac{dx}{\sqrt{6^2-(x-3)^2}}=-\frac{1}{\cancel{2}}\frac{u^{1/2}}{1/\cancel{2}}+3\operatorname{arcsen}\frac{x-3}{6}+c$$

$$=-u^{1/2}+3\operatorname{arcsen}\frac{x-3}{6}+c=-\sqrt{27+6x-x^2}+3\operatorname{arcsen}\frac{x-3}{6}+c$$

5.21.- $\int\frac{(x-1)dx}{3x^2-4x+3}$

Solución.- Sea: $u=3x^2-4x+3, du=(6x-4)dx$

$$\int\frac{(x-1)dx}{3x^2-4x+3}=\frac{1}{6}\int\frac{(6x-6)dx}{3x^2-4x+3}=\frac{1}{6}\int\frac{(6x-4-2)dx}{3x^2-4x+3}=\frac{1}{6}\int\frac{(6x-4)dx}{3x^2-4x+3}-\frac{1}{3}\int\frac{dx}{3x^2-4x+3}$$

$$=\frac{1}{6}\int\frac{du}{u}-\frac{1}{3}\int\frac{dx}{3x^2-4x+3}=\frac{1}{6}\int\frac{du}{u}-\frac{1}{3}\int\frac{dx}{3(x^2-\frac{4}{3}x+1)}$$

$$=\frac{1}{6}\int\frac{du}{u}-\frac{1}{9}\int\frac{dx}{(x^2-\frac{4}{3}x+1)}$$

Completando cuadrados se tiene:

$$x^2-\frac{4}{3}x+1=(x^2-\frac{4}{3}x+\frac{4}{9})+1-\frac{4}{9}=(x^2-\frac{4}{3}x+\frac{4}{9})+\frac{5}{9}=(x-\frac{2}{3})^2+(\frac{\sqrt{5}}{3})^2$$

$$=\frac{1}{6}\int\frac{du}{u}-\frac{1}{9}\int\frac{dx}{(x-\frac{2}{3})^2+(\frac{\sqrt{5}}{3})^2}=\frac{1}{6}\ell\eta|u|-\frac{1}{9}\frac{1}{\frac{\sqrt{5}}{3}}\operatorname{arc}\tau g\frac{x-\frac{2}{3}}{\frac{\sqrt{5}}{3}}+c$$

$$=\frac{1}{6}\ell\eta\left|3x^2-4x+3\right|-\frac{\sqrt{5}}{15}\operatorname{arc}\tau g\frac{3x-2}{\sqrt{5}}+c$$

5.22.- $\int\frac{(2x-3)dx}{x^2+6x+15}$

Solución.- Sea: $u=x^2+6x+15, du=(2x+6)dx$

$$\int\frac{(2x-3)dx}{x^2+6x+15}=\int\frac{(2x+6-9)dx}{x^2+6x+15}=\int\frac{(2x+6)dx}{x^2+6x+15}-9\int\frac{dx}{x^2+6x+15}$$

$=\int\frac{du}{u}-9\int\frac{dx}{x^2+6x+15}$, Completando cuadrados se tiene:

$$x^2+6x+15=(x^2+6x+9)+15-9=(x+3)^2+6^2=(x+3)^2+(\sqrt{6})^2$$

$$=\int\frac{du}{u}-9\int\frac{dx}{(x+3)^2+(\sqrt{6})^2}=\ell\eta\left|x^2+6x+15\right|-9\frac{1}{\sqrt{6}}\operatorname{arc}\tau g\frac{x+3}{\sqrt{6}}+c$$

$$=\ell\eta\left|x^2+6x+15\right|-\frac{3\sqrt{6}}{2}\operatorname{arc}\tau g\frac{x+3}{\sqrt{6}}+c$$

5.23.- $\int\frac{dx}{4x^2+4x+10}$

Solución.-

$\int\frac{dx}{4x^2+4x+10}=\int\frac{dx}{4(x^2+x+\frac{5}{2})}=\frac{1}{4}\int\frac{dx}{(x^2+x+\frac{5}{2})}$, Completando cuadrados:

$$x^2+x+\frac{5}{2}=(x^2+x+\frac{1}{4})+\frac{5}{2}-\frac{1}{4}=(x+\frac{1}{2})^2+\frac{9}{4}=(x+\frac{1}{2})^2+(\frac{3}{2})^2$$

$$=\frac{1}{4}\int\frac{dx}{(x+\frac{1}{2})^2+(\frac{3}{2})^2}=\frac{1}{4}\frac{1}{\frac{3}{2}}\operatorname{arc}\tau g\frac{x+\frac{1}{2}}{\frac{3}{2}}+c=\frac{1}{6}\operatorname{arc}\tau g\frac{2x+1}{3}+c$$

5.24.- $\int\frac{(2x+2)dx}{x^2-4x+9}$

Solución.- Sea: $u=x^2-4x+9, du=(2x-4)dx$

$$\int \frac{(2x+2)dx}{x^2-4x+9} = \int \frac{(2x-4+6)dx}{x^2-4x+9} = \int \frac{(2x-4)dx}{x^2-4x+9} + 6\int \frac{dx}{x^2-4x+9}$$

$= \int \frac{du}{u} + 6\int \frac{dx}{x^2-4x+9}$, Completando cuadrados se tiene:

$x^2-4x+9 = (x^2-4x+4)+9-4 = (x-2)^2+5 = (x-2)^2+(\sqrt{5})^2$,

$$= \int \frac{du}{u} + 6\int \frac{dx}{(x-2)^2+(\sqrt{5})^2} = \ell\eta|u| + 6\frac{1}{\sqrt{5}}\operatorname{arc}\tau g\,\frac{x-2}{\sqrt{5}} + c$$

$$= \ell\eta\left|x^2-4x+9\right| + \frac{6\sqrt{5}}{5}\operatorname{arc}\tau g\,\frac{x-2}{\sqrt{5}} + c$$

5.25.- $\int \frac{(2x+4)dx}{\sqrt{4x-x^2}}$

Solución.- Sea: $u = 4x-x^2+9, du = (4-2x)dx$

$$\int \frac{(2x+4)dx}{\sqrt{4x-x^2}} = -\int \frac{(-2x-4)dx}{\sqrt{4x-x^2}} = -\int \frac{(-2x+4-8)dx}{\sqrt{4x-x^2}} = -\int \frac{(-2x+4)dx}{\sqrt{4x-x^2}} + 8\int \frac{dx}{\sqrt{4x-x^2}}$$

$= -\int u^{-1/2}du + 8\int \frac{dx}{\sqrt{4x-x^2}}$, Completando cuadrados se tiene:

$4x-x^2 = -(x^2-4x) = -(x^2-4x+4-4) = -(x^2-4x+4)+4 = 2^2-(x-2)^2$

$$= -\int u^{-1/2}du + 8\int \frac{dx}{\sqrt{2^2-(x-2)^2}} = -2u^{1/2} + 8\operatorname{arcs}e\operatorname{n}\frac{x-2}{2} + c$$

$$= -2\sqrt{4x-x^2} + 8\operatorname{arcs}e\operatorname{n}\frac{x-2}{2} + c$$

5.26.- $\frac{2}{3}\int \frac{(x+3/2)dx}{9x^2-12x+8}$

Solución.- Sea: $u = 9x^2-12x+8, du = (18x-12)dx$

$$\frac{2}{3}\int \frac{(x+3/2)dx}{9x^2-12x+8} = \frac{2}{3}\frac{1}{18}\int \frac{(18x+27)dx}{9x^2-12x+8} = \frac{1}{27}\int \frac{(18x+27)dx}{9x^2-12x+8} = \frac{1}{27}\int \frac{(18x-12+39)dx}{9x^2-12x+8}$$

$$= \frac{1}{27}\int \frac{(18x-12)dx}{9x^2-12x+8} + \frac{39}{27}\int \frac{dx}{9x^2-12x+8} = \frac{1}{27}\int \frac{du}{u} + \frac{39}{27}\int \frac{dx}{9(x^2-\frac{4}{3}x+\frac{8}{9})}$$

$$= \frac{1}{27}\int \frac{du}{u} + \frac{39}{27\times 9}\int \frac{dx}{(x^2-\frac{4}{3}x+\frac{8}{9})}$$

Completando cuadrados se tiene:

$$x^2-\frac{4}{3}+\frac{8}{9} = (x^2-\frac{4}{3}x+\frac{4}{9})+\frac{8}{9}-\frac{4}{9} = (x-2/3)^2+4/9 = (x-2/3)^2+(2/3)^2$$

$$= \frac{1}{27}\int \frac{du}{u} + \frac{39}{27\times 9}\int \frac{dx}{(x-2/3)^2+(2/3)^2} = \frac{1}{27}\ell\eta|u| + \frac{39}{27\times 9}\frac{1}{2/3}\operatorname{arc}\tau g\,\frac{x-2/3}{2/3} + c$$

$$=\frac{1}{27}\ell\eta\left|9x^2-12x+8\right|-\frac{13}{54}\operatorname{arc}\tau g\frac{3x-2}{2}+c$$

5.27.- $\int\frac{(x+6)dx}{\sqrt{5-4x-x^2}}$

Solución.- Sea: $u=5-4x-x^2, du=(-4-2x)dx$

$$\int\frac{(x+6)dx}{\sqrt{5-4x-x^2}}=-\frac{1}{2}\int\frac{(-2x-12)dx}{\sqrt{5-4x-x^2}}=-\frac{1}{2}\int\frac{(-2x-4-8)dx}{\sqrt{5-4x-x^2}}$$

$$=-\frac{1}{2}\int\frac{(-2x-4)dx}{\sqrt{5-4x-x^2}}+4\int\frac{dx}{\sqrt{5-4x-x^2}}=-\frac{1}{2}\int\frac{du}{\sqrt{u}}+4\int\frac{dx}{\sqrt{5-4x-x^2}}$$

Completando cuadrados se tiene: $5-4x-x^2=9-(x+2)^2=3^2-(x+2)^2$

$$=-\frac{1}{2}\int\frac{du}{\sqrt{u}}+4\int\frac{dx}{\sqrt{3^2-(x+2)^2}}=-\sqrt{u}+4\operatorname{arcsen}\frac{x+2}{3}+c$$

$$=-\sqrt{5-4x-x^2}+4\operatorname{arcsen}\frac{x+2}{3}+c$$

5.28.- $\int\frac{dx}{2x^2+20x+60}$

Solución.-

$\int\frac{dx}{2x^2+20x+60}=\frac{1}{2}\int\frac{dx}{x^2+10x+30}$, Completando cuadrados se tiene:

$$x^2+10x+30=(x^2+10x+25)+5=(x+5)^2+(\sqrt{5})^2$$

$$=\frac{1}{2}\int\frac{dx}{(x+5)^2+(\sqrt{5})^2}=\frac{1}{2}\frac{1}{\sqrt{5}}\operatorname{arc}\tau g\frac{x+5}{\sqrt{5}}+c=\frac{\sqrt{5}}{10}\operatorname{arc}\tau g\frac{x+5}{\sqrt{5}}+c$$

5.29.- $\int\frac{3dx}{\sqrt{80+32x-4x^2}}$

Solución.-

$$\int\frac{3dx}{\sqrt{80+32x-4x^2}}=\int\frac{3dx}{\sqrt{4(20+8x-x^2)}}=\frac{3}{2}\int\frac{dx}{\sqrt{(20+8x-x^2)}}$$

Completando cuadrados se tiene:

$$20+8x-x^2=-(x^2-8x-20)=-(x^2-8x+16-20-16)=-(x^2-8x+16)+36$$

$$=-(x-4)^2+6^2=6^2-(x-4)^2$$

$$=\frac{3}{2}\int\frac{dx}{\sqrt{6^2-(x-4)^2}}=\frac{3}{2}\operatorname{arcsen}\frac{x-4}{6}+c$$

5.30.- $\int\frac{dx}{\sqrt{12x-4x^2-8}}$

Solución.-

$$\int\frac{dx}{\sqrt{12x-4x^2-8}}=\int\frac{dx}{\sqrt{4(-x^2+3x-2)}}=\frac{1}{2}\int\frac{dx}{\sqrt{(-x^2+3x-2)}}$$

Completando cuadrados se tiene:

$$-x^2+3x-2=-(x^2-3x+2)=-(x^2-3x+\frac{9}{4}+2-\frac{9}{4})=-(x^2-3x+\frac{9}{4})+\frac{1}{4}$$

$$=(\tfrac{1}{2})^2-(x-\tfrac{3}{2})^2$$

$$=\frac{1}{2}\int\frac{dx}{\sqrt{(\tfrac{1}{2})^2-(x-\tfrac{3}{2})^2}}=\frac{1}{2}\operatorname{arcsen}\frac{x-\tfrac{3}{2}}{\tfrac{1}{2}}+c=\frac{1}{2}\operatorname{arcsen}(2x-3)+c$$

5.31.- $\int\frac{5dx}{\sqrt{28-12x-x^2}}$

Solución.-

$\int\frac{5dx}{\sqrt{28-12x-x^2}}=5\int\frac{dx}{\sqrt{28-12x-x^2}}$, Completando cuadrados se tiene:

$$28-12x-x^2=8^2-(x+6)^2$$

$$=5\int\frac{dx}{\sqrt{8^2-(x+6)^2}}=5\operatorname{arcsen}\frac{x+6}{8}+c$$

5.32.- $\int\sqrt{12-8x-4x^2}\,dx$

Solución.- Sea: $u=x+1, du=dx; a=2$

$$\int\sqrt{12-8x-4x^2}\,dx=\int\sqrt{4(3-2x-x^2)}\,dx=2\int\sqrt{3-2x-x^2}\,dx$$

Completando cuadrados se tiene:

$$3-2x-x^2=-(x^2+2x-3)=-(x^2+2x+1)+4=2^2-(x+1)^2$$

$$2\int\sqrt{2^2-(x+1)^2}\,dx=2\int\sqrt{a^2-u^2}\,du=2(\frac{1}{2}u\sqrt{a^2-u^2}+\frac{a^2}{2}\operatorname{arcsen}\frac{u}{a})+c$$

$$=(x+1)\sqrt{-x^2-2x+3}+4\operatorname{arcsen}\frac{x+1}{2}+c$$

5.33.- $\sqrt{x^2-x+\tfrac{5}{4}}\,dx$

Solución.- Sea: $u=x-\tfrac{1}{2}, du=dx; a=1$

Completando cuadrados se tiene:

$$x^2-x+\tfrac{5}{4}=(x-\tfrac{1}{2})^2+1$$

$$\sqrt{x^2-x+\tfrac{5}{4}}\,dx=\sqrt{(x-\tfrac{1}{2})^2+1}\,dx=\sqrt{u^2+a^2}\,du$$

$$=\frac{1}{2}u\sqrt{u^2+a^2}+\frac{1}{2}a^2\ell\eta\left|u+\sqrt{u^2+a^2}\right|+c$$

$$=\frac{1}{2}(x-\tfrac{1}{2})\sqrt{x^2-x+\tfrac{5}{4}}+\frac{1}{2}\ell\eta\left|x-\tfrac{1}{2}+\sqrt{x^2-x+\tfrac{5}{4}}\right|+c$$

$$=\frac{1}{4}(2x-1)\sqrt{x^2-x+\tfrac{5}{4}}+\frac{1}{2}\ell\eta\left|x-\tfrac{1}{2}+\sqrt{x^2-x+\tfrac{5}{4}}\right|+c$$

5.34.- $\int\frac{dx}{x^2-2x+5}$

Solución.- Completando cuadrados se tiene:

$$x^2-2x+5=(x^2-2x+4)+1=(x-2)^2+1$$

$$\int\frac{dx}{x^2-2x+5}=\int\frac{dx}{(x-2)^2+1}=\operatorname{arc}\tau g(x-2)+c$$

5.35.- $\int\frac{(1-x)dx}{\sqrt{8+2x-x^2}}$

Solución.- Sea: $u=8+2x-x^2, du=(2-2x)dx=2(1-x)dx$

$$\int\frac{(1-x)dx}{\sqrt{8+2x-x^2}}=\frac{1}{2}\int\frac{du}{\sqrt{u}}=\frac{1}{2}\int u^{-1/2}du=\sqrt{u}+c=\sqrt{8+2x-x^2}+c$$

5.36.- $\int\frac{xdx}{x^2+4x+5}$

Solución.- Sea: $u=x^2+4x+5, du=(2x+4)dx$

$$\int\frac{xdx}{x^2+4x+5}=\frac{1}{2}\int\frac{2xdx}{x^2+4x+5}=\frac{1}{2}\int\frac{(2x+4)-4}{x^2+4x+5}dx$$

$=\frac{1}{2}\int\frac{(2x+4)dx}{x^2+4x+5}-2\int\frac{dx}{x^2+4x+5}=\frac{1}{2}\int\frac{du}{u}-2\int\frac{dx}{x^2+4x+5}$, Completando cuadrados se tiene: $x^2+4x+5=(x^2+4x+4)+1=(x+2)^2+1$

$$=\frac{1}{2}\int\frac{du}{u}-2\int\frac{dx}{(x+2)^2+1}=\frac{1}{2}\ell\eta|u|-2\operatorname{arc}\tau g(x+2)+c$$

$$=\frac{1}{2}\ell\eta\left|x^2+4x+5\right|-2\operatorname{arc}\tau g(x+2)+c$$

5.37.- $\int\frac{(2x+3)dx}{4x^2+4x+5}$

Solución.- Sea: $u=4x^2+4x+5, du=(8x+4)dx$

$$\int\frac{(2x+3)dx}{4x^2+4x+5}=\frac{1}{4}\int\frac{(8x+12)dx}{4x^2+4x+5}=\frac{1}{4}\int\frac{(8x+4)+8}{4x^2+4x+5}dx$$

$$\frac{1}{4}\int\frac{(8x+4)dx}{4x^2+4x+5}+2\int\frac{dx}{4x^2+4x+5}=\frac{1}{4}\int\frac{du}{u}+2\int\frac{dx}{4x^2+4x+5}=\frac{1}{4}\int\frac{du}{u}+2\int\frac{dx}{4(x^2+x+5/4)}$$

$=\frac{1}{4}\int\frac{du}{u}+\frac{1}{2}\int\frac{dx}{(x^2+x+5/4)}$, Completando cuadrados se tiene:

$$x^2+x+\frac{5}{4}=(x^2+x+\frac{1}{4})+1=(x+1/2)^2+1$$

$$=\frac{1}{4}\int\frac{du}{u}+\frac{1}{2}\int\frac{dx}{(x+1/2)^2+1}=\frac{1}{4}\ell\eta|u|+\frac{1}{2}\operatorname{arc}\tau g(x+1/2)+c$$

5.38.- $\int\frac{(x+2)dx}{x^2+2x+2}$

Solución.- Sea: $u=x^2+2x+2, du=(2x+2)dx$

$$\int \frac{(x+2)dx}{x^2+2x+2} = \frac{1}{2}\int \frac{(2x+4)dx}{x^2+2x+2} = \frac{1}{2}\int \frac{(2x+2)+2}{x^2+2x+2}dx = \frac{1}{2}\int \frac{(2x+2)dx}{x^2+2x+2} + \int \frac{dx}{x^2+2x+2}$$
$$= \frac{1}{2}\int \frac{du}{u} + \int \frac{dx}{x^2+2x+2} = \frac{1}{2}\int \frac{du}{u} + \int \frac{dx}{(x+1)^2+1}$$
$$= \frac{1}{2}\ell\eta|u| + \operatorname{arc}\tau g(x+1) + c = \frac{1}{2}\ell\eta\left|x^2+2x+2\right| + \operatorname{arc}\tau g(x+1) + c$$

5.39.- $\int \frac{(2x+1)dx}{x^2+8x-2}$

Solución.- Sea: $u = x^2+8x-2, du = (2x+8)dx$

$$\int \frac{(2x+1)dx}{x^2+8x-2} = \int \frac{(2x+8)-7dx}{x^2+8x-2} = \int \frac{(2x+8)dx}{x^2+8x-2} - 7\int \frac{dx}{x^2+8x-2}$$
$$= \int \frac{du}{u} - 7\int \frac{dx}{(x^2+8x+16)-18} = \int \frac{du}{u} - 7\int \frac{dx}{(x+4)^2-(3\sqrt{2})^2}$$
$$= \ell\eta|u| - 7\frac{1}{2(3\sqrt{2})}\ell\eta\left|\frac{(x+4)-(3\sqrt{2})}{(x+4)+(3\sqrt{2})}\right| + c$$
$$= \ell\eta\left|x^2+8x-2\right| - \frac{7\sqrt{2}}{12}\ell\eta\left|\frac{(x+4)-(3\sqrt{2})}{(x+4)+(3\sqrt{2})}\right| + c$$

5.40.- $\int \frac{dx}{\sqrt{-x^2-6x}}$

Solución.- Completando cuadrados se tiene:

$$-x^2-6x = -(x^2+6x) = -(x^2+6x+9)+9 = 3^2-(x+3)^2$$
$$\int \frac{dx}{\sqrt{3^2-(x+3)^2}} = \operatorname{arcs}e\operatorname{n}\frac{x+3}{3} + c$$

5.41.- $\int \frac{(x-1)dx}{x^2+2x+2}$

Solución.- Sea: $u = x^2+2x+2, du = (2x+2)dx$

$$\int \frac{(x-1)dx}{x^2+2x+2} = \frac{1}{2}\int \frac{(2x+2)-4}{x^2+2x+2}dx = \frac{1}{2}\int \frac{(2x+2)dx}{x^2+2x+2} - 2\int \frac{dx}{x^2+2x+2}$$
$$= \frac{1}{2}\int \frac{du}{u} - 2\int \frac{dx}{x^2+2x+2} = \frac{1}{2}\int \frac{du}{u} - 2\int \frac{dx}{(x+1)^2+1} = \frac{1}{2}\ell\eta|u| - 2\operatorname{arc}\tau g(x+1) + c$$
$$= \frac{1}{2}\ell\eta\left|x^2+2x+2\right| - 2\operatorname{arc}\tau g(x+1) + c$$