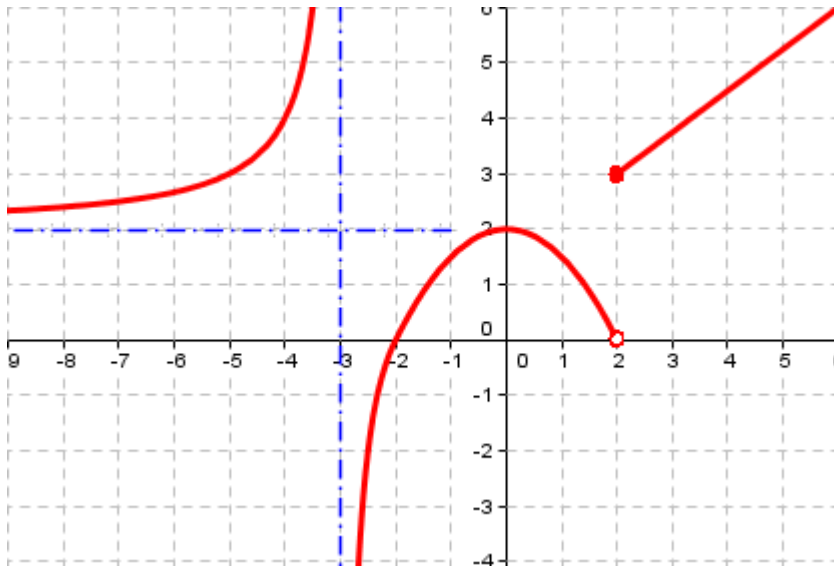


EXAMEN DE LÍMITES

1. Sobre la gráfica de la función $f(x)$, halla:



- | | |
|--|--|
| a) $\lim_{x \rightarrow -3^-} f(x)$ | b) $\lim_{x \rightarrow -3^+} f(x)$ |
| c) $\lim_{x \rightarrow 0} f(x)$ | d) $\lim_{x \rightarrow -2} f(x)$ |
| e) $\lim_{x \rightarrow +\infty} f(x)$ | f) $\lim_{x \rightarrow -\infty} f(x)$ |
| g) $\lim_{x \rightarrow 2^-} f(x)$ | h) $\lim_{x \rightarrow 2^+} f(x)$ |
| i) $\lim_{x \rightarrow -4} f(x)$ | j) $\lim_{x \rightarrow 1} f(x)$ |

2. Dadas las funciones:

$$f(x) = \frac{x^2 - 2x}{x^3 + x}$$

$$g(x) = \frac{x^2 - 5}{1 + x}$$

Hallar:

- | | | | | | |
|---------------------------------------|--|---|---|---|---|
| a) $\lim_{x \rightarrow \infty} f(x)$ | b) $\lim_{x \rightarrow +\infty} g(x)$ | c) $\lim_{x \rightarrow +\infty} f(x) \cdot g(x)$ | d) $\lim_{x \rightarrow +\infty} f(x) + g(x)$ | e) $\lim_{x \rightarrow +\infty} f(x) - g(x)$ | f) $\lim_{x \rightarrow +\infty} \frac{f(x)}{g(x)}$ |
|---------------------------------------|--|---|---|---|---|

3. Sea la función definida a trozos:

$$f(x) = \begin{cases} x^2 - 3 & \text{si } x < 2 \\ x - 2 & \text{si } x \geq 2 \end{cases}$$

- a) Representa gráficamente la función.
 b) Calcular, si existe, $\lim_{x \rightarrow 2} f(x)$.

4. Calcular los siguientes límites:

1) $\lim_{x \rightarrow +\infty} \frac{x^2 - 2x}{x^3 + x}$

2) $\lim_{x \rightarrow +\infty} \left(\frac{2x-1}{3x+2} \right)^x$

3) $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 9}$

4) $\lim_{x \rightarrow +\infty} \left(2 - \frac{1}{x^2 + 1} \right)^{3-x}$

5) $\lim_{x \rightarrow 2} \left(\frac{x+2}{x-1} - \frac{2x+4}{x^2-1} \right)$

6) $\lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^2 - 4x + 3}$

7) $\lim_{x \rightarrow +\infty} \left(\frac{4x-2}{3x+5} \right)^{x^2+1}$

8) $\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{4x}{x^2-9} \right)$

9) $\lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x^2 - 7x + 12}$

10) $\lim_{x \rightarrow +\infty} \frac{3 - \sqrt{x-2}}{\sqrt{2x+1}}$

11) $\lim_{x \rightarrow +\infty} \frac{3x+2}{\sqrt{x^2 - 3x + 1}}$

12) $\lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x^2-4}}$

13) $\lim_{x \rightarrow 1} \frac{\sqrt{x^3-1}}{\sqrt{x^2-1}}$

14) $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3}$

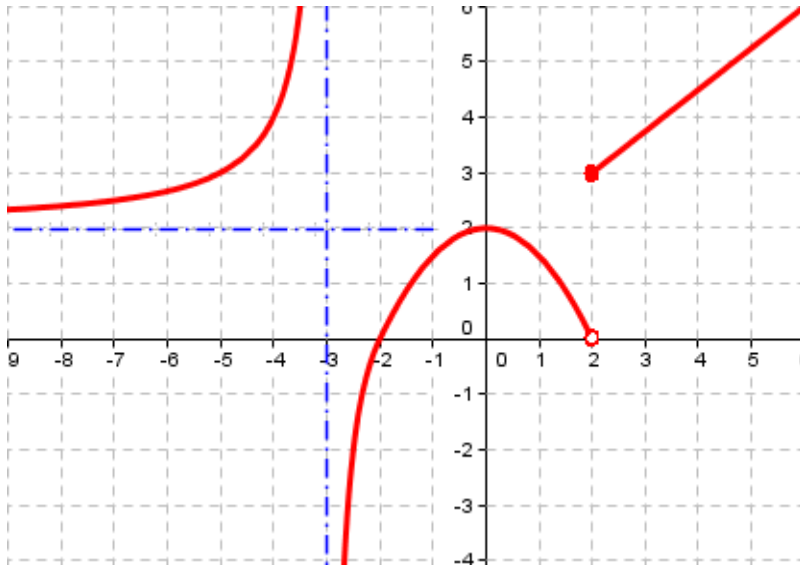
15) $\lim_{x \rightarrow -\infty} \frac{x^2 + 2x - 1}{\sqrt{1-x^3}}$

16) $\lim_{x \rightarrow 0} \frac{\sqrt{1-x} - \sqrt{x+1}}{x}$

17) $\lim_{x \rightarrow +\infty} (\sqrt{x^2+1} - x)$

18) $\lim_{x \rightarrow +\infty} (\sqrt{x^2-9} - \sqrt{x^2-6x})$

1. Sobre la gráfica de la función $f(x)$, halla:



- a) $\lim_{x \rightarrow -3^-} f(x) = +\infty$
- b) $\lim_{x \rightarrow -3^+} f(x) = -\infty$
- c) $\lim_{x \rightarrow 0} f(x) = 2$
- d) $\lim_{x \rightarrow -2} f(x) = 0$
- e) $\lim_{x \rightarrow +\infty} f(x) = +\infty$
- f) $\lim_{x \rightarrow -\infty} f(x) = 2$
- g) $\lim_{x \rightarrow 2^-} f(x) = 0$
- h) $\lim_{x \rightarrow 2^+} f(x) = 3$
- i) $\lim_{x \rightarrow -4} f(x) = 4$
- j) $\lim_{x \rightarrow 1} f(x) = 1,5$

2. Dadas las funciones

$$f(x) = \frac{x^2 - 2x}{x^3 + x}$$

$$g(x) = \frac{x^2 - 5}{1 + x}$$

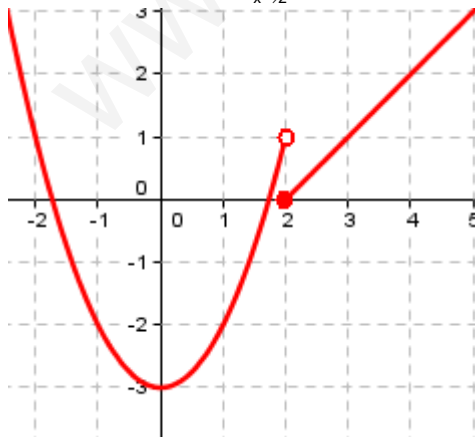
Hallar:

- a) $\lim_{x \rightarrow \infty} f(x)$
 - b) $\lim_{x \rightarrow +\infty} g(x)$
 - c) $\lim_{x \rightarrow +\infty} f(x) \cdot g(x)$
 - d) $\lim_{x \rightarrow +\infty} f(x) + g(x)$
 - e) $\lim_{x \rightarrow +\infty} f(x) - g(x)$
 - f) $\lim_{x \rightarrow +\infty} \frac{f(x)}{g(x)}$
-
- a) $\lim_{x \rightarrow \infty} f(x) = 0$
 - b) $\lim_{x \rightarrow +\infty} g(x) = +\infty$
 - c) $\lim_{x \rightarrow +\infty} f(x) \cdot g(x) = 1$
 - d) $\lim_{x \rightarrow +\infty} f(x) + g(x) = +\infty$
 - e) $\lim_{x \rightarrow +\infty} f(x) - g(x) = -\infty$
 - f) $\lim_{x \rightarrow +\infty} \frac{f(x)}{g(x)} = 0$

3. Sea la función definida a trozos:

$$f(x) = \begin{cases} x^2 - 3 & \text{si } x < 2 \\ x - 2 & \text{si } x \geq 2 \end{cases}$$

- a) Representa gráficamente la función.
- b) Calcular, si existe, $\lim_{x \rightarrow 2} f(x)$.



$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2} (x^2 - 3) = 1$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} (x - 2) = 0$$

Al no coincidir los límites laterales, no existe $\lim_{x \rightarrow 2} f(x)$

4. Calcular los siguientes límites:

$$1) \lim_{x \rightarrow +\infty} \frac{x^2 - 2x}{x^3 + x} = 0$$

$$2) \lim_{x \rightarrow +\infty} \left(\frac{2x-1}{3x+2} \right)^x = \left(\frac{2}{3} \right)^{+\infty} = 0$$

$$3) \lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 9} = \frac{0}{0} = \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x^2 + 3x + 9)}{\cancel{(x-3)}(x+3)} = \lim_{x \rightarrow 3} \frac{x^2 + 3x + 9}{x+3} = \frac{27}{6} = \frac{9}{2}$$

$$4) \lim_{x \rightarrow +\infty} \left(2 - \frac{1}{x^2 + 1} \right)^{3-x} = 2^{-\infty} = \left(\frac{1}{2} \right)^{+\infty} = 0$$

$$5) \lim_{x \rightarrow 2} \left(\frac{x+2}{x-1} - \frac{2x+4}{x^2-1} \right) = \infty - \infty = \lim_{x \rightarrow 2} \frac{x^2 + 3x + 2 - 2x - 4}{x^2 - 1} = \lim_{x \rightarrow 2} \frac{x^2 + x - 2}{x^2 - 1} = \lim_{x \rightarrow 2} \frac{(x+2)(x-1)}{(x-1)(x+1)} = \frac{3}{2}$$

$$6) \lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^2 - 4x + 3} = \frac{0}{0} = \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}^2(x-2)}{\cancel{(x-1)}(x-3)} = \lim_{x \rightarrow 1} \frac{(x-2)(x-1)}{x-3} = 0$$

$$7) \lim_{x \rightarrow +\infty} \left(\frac{4x-2}{3x+5} \right)^{x^2+1} = \left(\frac{4}{3} \right)^{+\infty} = +\infty$$

$$8) \lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{4x}{x^2-9} \right) = \infty - \infty = \lim_{x \rightarrow 3} \frac{x+3-4x}{x^2-9} = \lim_{x \rightarrow 3} \frac{3-3x}{x^2-9} = \begin{cases} -\infty & \text{si } x \rightarrow 3^+ \\ +\infty & \text{si } x \rightarrow 3^- \end{cases}$$

$$9) \lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x^2 - 7x + 12} = \frac{0}{0} = \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x-3)}{\cancel{(x-3)}(x-4)} = \lim_{x \rightarrow 3} \frac{x-3}{x-4} = \frac{0}{-1} = 0$$

$$10) \lim_{x \rightarrow +\infty} \frac{3 - \sqrt{x-2}}{\sqrt{2x+1}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$11) \lim_{x \rightarrow 1} \frac{\sqrt{x^3-1}}{\sqrt{x^2-1}} = \frac{0}{0} = \lim_{x \rightarrow 1} \frac{\sqrt{(x-1)(x^2+x+1)}}{\sqrt{(x-1)(x+1)}} = \lim_{x \rightarrow 1} \sqrt{\frac{x^2+x+1}{x+1}} = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$$

$$12) \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x^2-4}} = \frac{0}{0} = \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{(x-2)(x+2)}} = \lim_{x \rightarrow 2} \frac{\sqrt{x-2}}{\sqrt{x+2}} = \frac{0}{2} = 0$$

$$13) \lim_{x \rightarrow +\infty} \frac{3x+2}{\sqrt{x^2-3x+1}} = 3$$

$$14) \lim_{x \rightarrow 3} \frac{\sqrt{x+1}-2}{x-3} = \frac{0}{0} = \lim_{x \rightarrow 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x \rightarrow 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$$

$$15) \lim_{x \rightarrow -\infty} \frac{x^2 + 2x - 1}{\sqrt{1-x^3}} = +\infty$$

$$16) \lim_{x \rightarrow 0} \frac{\sqrt{1-x} - \sqrt{x+1}}{x} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{(1-x-x-1)}{x(\sqrt{1-x} + \sqrt{x+1})} = \lim_{x \rightarrow 0} \frac{-2}{(\sqrt{1-x} + \sqrt{x+1})} = -1$$

$$17) \lim_{x \rightarrow +\infty} (\sqrt{x^2+1} - x) = \infty - \infty = \lim_{x \rightarrow +\infty} \frac{x^2+1-x^2}{(\sqrt{x^2+1}+x)} = \lim_{x \rightarrow +\infty} \frac{1}{(\sqrt{x^2+1}+x)} = 0$$

$$18) \lim_{x \rightarrow +\infty} (\sqrt{x^2-9} - \sqrt{x^2-6x}) = \infty - \infty = \lim_{x \rightarrow +\infty} \frac{x^2-9-x^2+6x}{(\sqrt{x^2-9} + \sqrt{x^2-6x})} = \lim_{x \rightarrow +\infty} \frac{6x-9}{(\sqrt{x^2-9} + \sqrt{x^2-6x})} = 3$$