

5. IDENTIDADES TRIGONOMÉTRICAS

EJERCICIOS PROPUESTOS

1. Demostrar las siguientes igualdades:

a) $\sec x + \operatorname{cosec} x = \operatorname{tg} x \cdot \operatorname{cosec} x + \operatorname{ctg} x \cdot \sec x$

b) $\operatorname{tg} x + \operatorname{ctg} x = \sec x \cdot \operatorname{cosec} x$

c) $\frac{1 - \operatorname{sen} x}{\cos x} = \frac{\operatorname{sen}(90^\circ - x)}{1 + \operatorname{sen} x}$

d) $\operatorname{sen}^4 x - \cos^4 x = \operatorname{sen}^2 x - \cos^2 x$

e) $\frac{\operatorname{ctg} x + 1}{\sec x + \operatorname{cosec} x} = \cos x$

f) $\frac{\operatorname{sen}\left(\frac{\pi}{2} - x\right) - \cos(\pi - x)}{\cos\left(\frac{\pi}{2} + x\right) + \operatorname{sen}(\pi + x)} = -\operatorname{ctg} x$

g) $\frac{\operatorname{tg} x}{\operatorname{tg} 2x - \operatorname{tg} x} = \cos 2x$

h) $\sec^2 x + \operatorname{cosec}^2 x = \sec^2 x \cdot \operatorname{cosec}^2 x$

$\sec^4 x - \sec^2 x = \operatorname{tg}^4 x + \operatorname{tg}^2 x$

Solución:

$$\text{a) } \sec x + \operatorname{cosec} x = \operatorname{tg} x \cdot \operatorname{cosec} x + \operatorname{ctg} x \cdot \sec x$$

$$\operatorname{tg} x \cdot \operatorname{cosec} x + \operatorname{ctg} x \cdot \sec x = \frac{\operatorname{sen} x}{\cos x} \cdot \frac{1}{\operatorname{sen} x} + \frac{\cos x}{\operatorname{sen} x} \cdot \frac{1}{\cos x} = \frac{1}{\cos x} + \frac{1}{\operatorname{sen} x} = \sec x + \operatorname{cosec} x$$

$$\text{b) } \operatorname{tg} x + \operatorname{ctg} x = \sec x \cdot \operatorname{cosec} x$$

$$\operatorname{tg} x + \operatorname{ctg} x = \frac{\operatorname{sen} x}{\cos x} + \frac{\cos x}{\operatorname{sen} x} = \frac{\operatorname{sen}^2 x + \cos^2 x}{\cos x \cdot \operatorname{sen} x} = \frac{1}{\cos x \cdot \operatorname{sen} x} = \frac{1}{\cos x} \cdot \frac{1}{\operatorname{sen} x} = \sec x \cdot \operatorname{cosec} x$$

$$\text{c) } \frac{1 - \operatorname{sen} x}{\cos x} = \frac{\operatorname{sen}(90^\circ - x)}{1 + \operatorname{sen} x}$$

$$\frac{1 - \operatorname{sen} x}{\cos x} = \frac{\operatorname{sen}(90^\circ - x)}{1 + \operatorname{sen} x} \rightarrow (1 - \operatorname{sen} x)(1 + \operatorname{sen} x) = \cos x \cdot \operatorname{sen}(90^\circ - x) \rightarrow 1 - \operatorname{sen}^2 x = \cos x \cdot \cos x \rightarrow$$

$$\cos^2 x = \cos^2 x$$

$$\text{d) } \operatorname{sen}^4 x - \cos^4 x = \operatorname{sen}^2 x - \cos^2 x$$

$$\operatorname{sen}^4 x - \cos^4 x = (\operatorname{sen}^2 x - \cos^2 x)(\operatorname{sen}^2 x + \cos^2 x) = \operatorname{sen}^2 x - \cos^2 x$$

$$\text{e) } \frac{\operatorname{ctg} x + 1}{\sec x + \operatorname{cosec} x} = \cos x$$

$$\frac{\operatorname{ctg} x + 1}{\sec x + \operatorname{cosec} x} = \frac{\frac{\cos x}{\operatorname{sen} x} + 1}{\frac{1}{\cos x} + \frac{1}{\operatorname{sen} x}} = \frac{\frac{\cos x + \operatorname{sen} x}{\operatorname{sen} x}}{\frac{\cos x + \operatorname{sen} x}{\operatorname{sen} x \cdot \cos x}} = \frac{\operatorname{sen} x \cdot \cos x}{\operatorname{sen} x} = \cos x$$

$$\text{f) } \frac{\operatorname{sen}\left(\frac{\pi}{2} - x\right) - \cos(\pi - x)}{\cos\left(\frac{\pi}{2} + x\right) + \operatorname{sen}(\pi + x)} = -\operatorname{ctg} x$$

Teniendo en cuenta:

$$\text{a) } \operatorname{sen}\left(\frac{\pi}{2} - x\right) = \cos x \quad \text{b) } \cos(\pi - x) = -\cos x \quad \text{c) } \operatorname{sen}(\pi + x) = -\operatorname{sen} x \quad \text{d) } \cos\left(\frac{\pi}{2} + x\right) = -\operatorname{sen} x$$

$$\frac{\operatorname{sen}\left(\frac{\pi}{2} - x\right) - \cos(\pi - x)}{\cos\left(\frac{\pi}{2} + x\right) + \operatorname{sen}(\pi + x)} = \frac{\cos x + \cos x}{-\operatorname{sen} x - \operatorname{sen} x} = \frac{2\cos x}{-2\operatorname{sen} x} = -\operatorname{ctg} x$$

$$g) \frac{\operatorname{tg} x}{\operatorname{tg} 2x - \operatorname{tg} x} = \cos 2x$$

$$\frac{\operatorname{tg} x}{\operatorname{tg} 2x - \operatorname{tg} x} = \frac{\operatorname{tg} x}{\frac{2\operatorname{tg} x}{1 - \operatorname{tg}^2 x} - \operatorname{tg} x} = \frac{\operatorname{tg} x}{\frac{\operatorname{tg} x + \operatorname{tg}^3 x}{1 - \operatorname{tg}^2 x}} = \frac{\operatorname{tg} x(1 - \operatorname{tg}^2 x)}{\operatorname{tg} x(1 + \operatorname{tg}^2 x)} = \frac{1 - \frac{\operatorname{sen}^2 x}{\cos^2 x}}{1 + \frac{\operatorname{sen}^2 x}{\cos^2 x}} = \frac{\cos^2 x - \operatorname{sen}^2 x}{\cos^2 x + \operatorname{sen}^2 x} = \cos 2x$$

$$h) \sec^2 x + \operatorname{cosec}^2 x = \sec^2 x \cdot \operatorname{cosec}^2 x$$

$$\sec^2 x + \operatorname{cosec}^2 x = \frac{1}{\cos^2 x} + \frac{1}{\operatorname{sen}^2 x} = \frac{\operatorname{sen}^2 x + \cos^2 x}{\cos^2 x \cdot \operatorname{sen}^2 x} = \frac{1}{\cos^2 x \cdot \operatorname{sen}^2 x} = \frac{1}{\cos^2 x} \cdot \frac{1}{\operatorname{sen}^2 x} = \sec^2 x \cdot \operatorname{cosec}^2 x$$

$$i) \sec^4 x - \sec^2 x = \operatorname{tg}^4 x + \operatorname{tg}^2 x$$

$$\begin{aligned} \sec^4 x - \sec^2 x &= \sec^2 x (\sec^2 x - 1) = \sec^2 x \left(\frac{1}{\cos^2 x} - 1 \right) = \sec^2 x \frac{1 - \cos^2 x}{\cos^2 x} = \frac{\operatorname{sen}^2 x}{\cos^4 x} = \frac{\operatorname{sen}^2 x}{\cos^2 x} \cdot \frac{1}{\cos^2 x} = \\ &= \operatorname{tg}^2 x \cdot \sec^2 x = \operatorname{tg}^2 x \cdot (\operatorname{tg}^2 x + 1) = \operatorname{tg}^4 x + \operatorname{tg}^2 x \end{aligned}$$