

5. IDENTIDADES TRIGONOMÉTRICAS

EJERCICIOS RESUELTOS

1. Demuestra las siguientes identidades:

a) $\cos x \cdot (1 + \operatorname{tg} x) = \operatorname{sen} x \cdot (1 + \operatorname{ctg} x)$

b) $\frac{\operatorname{sen}(-x) \cdot \operatorname{ctg}(2\pi - x)}{\operatorname{tg}(\pi + x) \cdot \operatorname{sen}\left(\frac{\pi}{2} - x\right)} = \operatorname{ctg} x$

Solución:

a) $\cos x \cdot (1 + \operatorname{tg} x) = \operatorname{sen} x \cdot (1 + \operatorname{ctg} x)$

$$\cos x \cdot \left(1 + \frac{\operatorname{sen} x}{\cos x}\right) = \operatorname{sen} x \cdot \left(1 + \frac{\cos x}{\operatorname{sen} x}\right) \rightarrow \cos x \cdot \left(\frac{\cos x + \operatorname{sen} x}{\cos x}\right) = \operatorname{sen} x \cdot \left(\frac{\operatorname{sen} x + \cos x}{\operatorname{sen} x}\right) \rightarrow$$

$$\cancel{\cos x} \cdot \left(\frac{\cos x + \operatorname{sen} x}{\cancel{\cos x}}\right) = \cancel{\operatorname{sen} x} \cdot \left(\frac{\operatorname{sen} x + \cos x}{\cancel{\operatorname{sen} x}}\right) \rightarrow \cos x + \operatorname{sen} x = \cos x + \operatorname{sen} x$$

b) $\frac{\operatorname{sen}(-x) \cdot \operatorname{ctg}(2\pi - x)}{\operatorname{tg}(\pi + x) \cdot \operatorname{sen}\left(\frac{\pi}{2} - x\right)} = \operatorname{ctg} x$

Teniendo en cuenta:

a) $\operatorname{sen}(-x) = -\operatorname{sen} x$ b) $\operatorname{ctg}(2\pi - x) = -\operatorname{ctg} x$ c) $\operatorname{tg}(\pi + x) = \operatorname{tg} x$ d) $\operatorname{sen}\left(\frac{\pi}{2} - x\right) = \cos x$

$$\frac{\operatorname{sen}(-x) \cdot \operatorname{ctg}(2\pi - x)}{\operatorname{tg}(\pi + x) \cdot \operatorname{sen}\left(\frac{\pi}{2} - x\right)} = \frac{(-\operatorname{sen} x) \cdot (-\operatorname{ctg} x)}{\operatorname{tg} x \cdot \cos x} = \frac{\cancel{\operatorname{sen} x} \cdot \frac{\cos x}{\cancel{\operatorname{sen} x}}}{\frac{\operatorname{sen} x}{\cancel{\cos x}} \cdot \cancel{\cos x}} = \frac{\cos x}{\operatorname{sen} x} = \operatorname{ctg} x$$

2. Demostrar las siguientes igualdades:

a) $\cos(2\pi - x) \cdot \operatorname{ctg} x + \operatorname{sen}(\pi - x) = \sec\left(\frac{\pi}{2} - x\right)$

b) $\sec x - \cos x = \operatorname{tg} x \cdot \operatorname{sen} x$

Solución:

a) $\cos(2\pi - x) \cdot \operatorname{ctg} x + \operatorname{sen}(\pi - x) = \sec\left(\frac{\pi}{2} - x\right)$

$$\cos(2\pi - x) \cdot \operatorname{ctg} x + \operatorname{sen}(\pi - x) = \cos x \cdot \frac{\cos x}{\operatorname{sen} x} + \operatorname{sen} x = \frac{\cos^2 x}{\operatorname{sen} x} + \operatorname{sen} x = \frac{\cos^2 x + \operatorname{sen}^2 x}{\operatorname{sen} x} = \frac{1}{\operatorname{sen} x}$$

$$\sec\left(\frac{\pi}{2} - x\right) = \frac{1}{\cos\left(\frac{\pi}{2} - x\right)} = \frac{1}{\operatorname{sen} x}$$

b) $\sec x - \cos x = \operatorname{tg} x \cdot \operatorname{sen} x$

$$\sec x - \cos x = \frac{1}{\cos x} - \cos x = \frac{1 - \cos^2 x}{\cos x} = \frac{\operatorname{sen}^2 x}{\cos x} = \operatorname{sen} x \cdot \frac{\operatorname{sen} x}{\cos x} = \operatorname{sen} x \cdot \operatorname{tg} x$$

3. Demuestra las siguientes identidades:

$$\text{a) } \cos x + \cos y = (\sin x - \sin y) \operatorname{ctg} \left(\frac{x-y}{2} \right)$$

$$\text{b) } \operatorname{tg} \frac{x}{2} = \frac{\sin x}{1 + \cos x}$$

Solución:

$$\text{a) } \cos x + \cos y = (\sin x - \sin y) \operatorname{ctg} \left(\frac{x-y}{2} \right)$$

Aplicando las fórmulas de adición:

$$\cos x + \cos y = 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$$

$$\sin x - \sin y = 2 \cos \left(\frac{x+y}{2} \right) \sin \left(\frac{x-y}{2} \right)$$

$$\cos x + \cos y = (\sin x - \sin y) \operatorname{ctg} \left(\frac{x-y}{2} \right) \rightarrow 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right) = 2 \cos \left(\frac{x+y}{2} \right) \sin \left(\frac{x-y}{2} \right) \operatorname{ctg} \left(\frac{x-y}{2} \right) \rightarrow$$

$$\cos \left(\frac{x-y}{2} \right) = \sin \left(\frac{x-y}{2} \right) \operatorname{ctg} \left(\frac{x-y}{2} \right) \rightarrow \frac{\cos \left(\frac{x-y}{2} \right)}{\sin \left(\frac{x-y}{2} \right)} = \operatorname{ctg} \left(\frac{x-y}{2} \right) \rightarrow \operatorname{ctg} \left(\frac{x-y}{2} \right) = \operatorname{ctg} \left(\frac{x-y}{2} \right)$$

$$\text{b) } \operatorname{tg} \frac{x}{2} = \frac{\sin x}{1 + \cos x}$$

Aplicando la fórmula del ángulo mitad:

$$\operatorname{tg} \frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$\operatorname{tg} \frac{x}{2} = \frac{\sqrt{1 - \cos x}}{\sqrt{1 + \cos x}} = \frac{\sqrt{1 - \cos x}}{\sqrt{1 + \cos x}} \cdot \frac{\sqrt{1 + \cos x}}{\sqrt{1 + \cos x}} = \frac{\sqrt{1 - \cos^2 x}}{1 + \cos x} = \frac{\sin x}{1 + \cos x}$$

4. Demostrar la siguiente igualdad:

$$\frac{\operatorname{tg} x}{\operatorname{tg} 2x - \operatorname{tg} x} = \cos 2x$$

Solución:

$$\frac{\operatorname{tg} x}{\operatorname{tg} 2x - \operatorname{tg} x} = \frac{\operatorname{tg} x}{\frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} - \operatorname{tg} x} = \frac{\operatorname{tg} x}{\frac{\operatorname{tg} x + \operatorname{tg}^3 x}{1 - \operatorname{tg}^2 x}} = \frac{\operatorname{tg} x (1 - \operatorname{tg}^2 x)}{\operatorname{tg} x (1 + \operatorname{tg}^2 x)} = \frac{1 - \frac{\operatorname{sen}^2 x}{\cos^2 x}}{1 + \frac{\operatorname{sen}^2 x}{\cos^2 x}} = \frac{\cos^2 x - \operatorname{sen}^2 x}{\cos^2 x + \operatorname{sen}^2 x} = \cos 2x$$