
29 Problemas Resueltos Ecuaciones Trigonometricas

1)

$$\operatorname{sen} x = 0$$

$$\operatorname{arcsen}(\operatorname{sen} x) = \operatorname{arcsen} 0 \quad f \circ f^{-1} = x$$

$$x = \operatorname{arcsen} 0 \Rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ k & x_1 = 0^\circ, 360^\circ, 720^\circ, \dots \\ x_2 = 180^\circ + 360^\circ k & x_2 = 180^\circ, 540^\circ, 900^\circ, \dots \end{cases}$$

$$x = 0^\circ + 180^\circ k$$

2) $\cos x = 0$

$$\operatorname{arccos}(\cos x) = \operatorname{arccos} 0 \quad f \circ f^{-1} = x$$

$$x = \operatorname{arccos} 0 \Rightarrow \begin{cases} x_1 = 90^\circ + 360^\circ k & x_1 = 90^\circ, 450^\circ, 810^\circ, \dots \\ x_2 = 270^\circ + 360^\circ k & x_2 = 270^\circ, 630^\circ, 990^\circ, \dots \end{cases}$$

$$x = 90^\circ + 180^\circ k$$

3) $\operatorname{tg} x = 0$

$$\operatorname{arctg}(\operatorname{tg} x) = \operatorname{arctg} 0 \quad f \circ f^{-1} = x$$

$$x = \operatorname{arctg} 0 \quad x = 0^\circ + 180^\circ k$$

4) $\operatorname{sen} x = 1$

$$\operatorname{arcsen}(\operatorname{sen} x) = \operatorname{arcsen} 1 \quad f \circ f^{-1} = x$$

$$x = \operatorname{arcsen} 1 \quad x = 90^\circ + 360^\circ k$$

5) $\cos x = 1$

$$\operatorname{arccos}(\cos x) = \operatorname{arccos} 1 \quad f \circ f^{-1} = x$$

$$x = \operatorname{arccos} 1 \quad x = 0^\circ + 360^\circ k$$

6) $\operatorname{tg} x = 1$

$$x = \operatorname{arctg} 1 \qquad x = 45^\circ + 180^\circ k$$

7) $\operatorname{sen} x = -1$

$$\operatorname{arcsen}(\operatorname{sen} x) = \operatorname{arcsen}(-1) \qquad f \circ f^{-1} = x$$

$$x = \operatorname{arcsen}(-1) \qquad x = 270^\circ + 360^\circ k$$

Problema N° 8: $\operatorname{cos} x = -1$

$$\operatorname{arccos}(\operatorname{cos} x) = \operatorname{arccos}(-1) \qquad f \circ f^{-1} = x$$

$$x = \operatorname{arccos}(-1) \qquad x = 180^\circ + 360^\circ k$$

9) $\operatorname{tg} x = -1$

$$\operatorname{arctg}(\operatorname{tg} x) = \operatorname{arctg}(-1) \qquad f \circ f^{-1} = x$$

$$x = \operatorname{arctg}(-1) \qquad x = 135^\circ + 180^\circ k$$

10) $\operatorname{sen} x = \frac{1}{2}$

$$\operatorname{arcsen}(\operatorname{sen} x) = \operatorname{arcsen}\left(\frac{1}{2}\right) \qquad f \circ f^{-1} = x$$

$$x = \operatorname{arcsen}\left(\frac{1}{2}\right) \Rightarrow \begin{cases} x_1 = 30^\circ + 360^\circ k \\ x_2 = 150^\circ + 360^\circ k \end{cases}$$

11) $\operatorname{sen} x = -\frac{1}{2}$

$$\operatorname{arcsen}(\operatorname{sen} x) = \operatorname{arcsen}\left(-\frac{1}{2}\right) \qquad f \circ f^{-1} = x$$

$$x = \operatorname{arcsen}\left(-\frac{1}{2}\right) \Rightarrow \begin{cases} x_1 = 210^\circ + 360^\circ k \\ x_2 = 330^\circ + 360^\circ k \end{cases}$$

12) $\operatorname{cos} x = \frac{1}{2}$

$$\operatorname{arccos}(\operatorname{cos} x) = \operatorname{arccos}\left(\frac{1}{2}\right) \qquad f \circ f^{-1} = x$$

$$x = \operatorname{arccos}\left(\frac{1}{2}\right) \Rightarrow \begin{cases} x_1 = 60^\circ + 360^\circ k \\ x_2 = 300^\circ + 360^\circ k \end{cases}$$

13) $\operatorname{cos} x = -\frac{1}{2}$

$$\operatorname{arccos}(\operatorname{cos} x) = \operatorname{arccos}\left(-\frac{1}{2}\right) \qquad f \circ f^{-1} = x$$

$$x = \arccos\left(-\frac{1}{2}\right) \Rightarrow \begin{cases} x_1 = 120^\circ + 360^\circ k \\ x_2 = 240^\circ + 360^\circ k \end{cases}$$

$$\operatorname{sen}\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} \rightarrow \begin{cases} \operatorname{sen} 60^\circ \\ \operatorname{sen} 120^\circ \end{cases}$$

$$x + 45^\circ = 60^\circ \quad x_1 = 15^\circ + 360^\circ k$$

$$x + 45^\circ = 120^\circ \quad x_2 = 75^\circ + 360^\circ k$$

14) $2\operatorname{tg} x - 3\operatorname{cotg} x - 1 = 0.$

$$2\operatorname{tg} x - \frac{3}{\operatorname{tg} x} - 1 = 0 \quad 2\operatorname{tg}^2 x - \operatorname{tg} x - 3 = 0$$

$$\operatorname{tg} x = \frac{1 \pm \sqrt{1+24}}{4} = \frac{1 \pm 5}{4}$$

$$\operatorname{tg} x = \frac{3}{2} \quad x = 56^\circ 18' 35'' + 180^\circ k$$

$$\operatorname{tg} x = -1 \quad x = 135^\circ + 180^\circ k$$

15) $3\operatorname{sen}^2 x - 5\operatorname{sen} x + 2 = 0$

$$\operatorname{sen} x = \frac{5 \pm \sqrt{25-24}}{6} = \frac{5 \pm 1}{6}$$

$$\operatorname{sen} x = 1 \quad x = 90^\circ + 360^\circ k$$

$$\operatorname{sen} x = \frac{2}{3} \quad x = \begin{cases} 41^\circ 48' 37'' + 360^\circ k \\ 138^\circ 11' 23'' + 360^\circ k \end{cases}$$

$$\text{sen}^2 x = \frac{1}{4} \quad \text{sen } x = \pm \frac{1}{2}$$

$$x = \text{arcsen } \frac{1}{2} \Rightarrow \begin{cases} x_1 = 30^\circ + 360^\circ k \\ x_2 = 150^\circ + 360^\circ k \end{cases}$$

$$x = \text{arcsen} \left(-\frac{1}{2} \right) \Rightarrow \begin{cases} x_3 = 210^\circ + 360^\circ k \\ x_4 = 330^\circ + 360^\circ k \end{cases}$$

17) $\cos 2x = 1 + 4\text{sen } x$

$$\cos^2 x - \text{sen}^2 x = 1 + 4\text{sen } x$$

$$1 - \text{sen}^2 x - \text{sen}^2 x = 1 + 4\text{sen } x$$

$$2\text{sen}^2 x + 4\text{sen } x = 0$$

$$2\text{sen } x (\text{sen } x + 2) = 0 \Rightarrow \begin{cases} \text{sen } x = 0 \\ \text{sen } x + 2 = 0 \end{cases}$$

$$x = \text{arcsen } 0 \Rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ k \\ x_2 = 180^\circ + 360^\circ k \end{cases}$$

$$x = 0^\circ + 180^\circ k$$

$$x = \text{arcsen}(-2) \quad \text{Sin solución}$$

18) $\text{sen}(2x + 60^\circ) + \text{sen}(x + 30^\circ) = 0$

$$2\text{sen} \left(\frac{3x}{2} + 45^\circ \right) \cos \left(\frac{x}{2} + 15^\circ \right) = 0$$

$$\text{sen} \left(\frac{3x}{2} + 45^\circ \right) = 0 \Rightarrow \begin{cases} \frac{3x}{2} + 45^\circ = 0^\circ + 360^\circ k & x = -30^\circ + 120^\circ k \\ \frac{3x}{2} + 45^\circ = 180^\circ + 360^\circ k & x = -30^\circ + 120^\circ k \end{cases}$$

$$\cos \left(\frac{x}{2} + 15^\circ \right) = 0 \Rightarrow \begin{cases} \frac{x}{2} + 15^\circ = 90^\circ + 360^\circ k & x = 150^\circ + 360^\circ k \\ \frac{x}{2} + 15^\circ = 270^\circ + 360^\circ k & x = 510^\circ + 360^\circ k \\ & x = 150^\circ + 360^\circ k \end{cases}$$

$$\operatorname{sen}^2 x - \cos^2 x = \frac{1}{2} \quad \cos^2 x - \operatorname{sen}^2 x = -\frac{1}{2} \quad \cos 2x = -\frac{1}{2}$$

$$2x = \begin{cases} 120^\circ + 360^\circ k \\ 240^\circ + 360^\circ k \end{cases} \Rightarrow x = \begin{cases} 60^\circ + 180^\circ k \\ 120^\circ + 180^\circ k \end{cases}$$

20) $\operatorname{tg} 2x = -\operatorname{tg} x$

$$\frac{2\operatorname{tg} x}{1 - \operatorname{tg}^2 x} = -\operatorname{tg} x$$

$$\operatorname{tg} x (\operatorname{tg}^2 x - 3) = 0$$

$$\operatorname{tg} x = 0 \quad x = 0^\circ + 180^\circ k$$

$$\operatorname{tg} x = \pm\sqrt{3} \quad \begin{cases} x = 60^\circ + 180^\circ k \\ x = 120^\circ + 180^\circ k \end{cases}$$

21) $\cos 8x + \cos 6x = 2 \cos 210^\circ \cdot \cos x$

$$2 \cos 7x \cdot \cos x = 2 \cos 210^\circ \cdot \cos x$$

$$\cos x (\cos 7x - \cos 210^\circ) = 0$$

$$\cos x = 0 \quad x = \begin{cases} x = 90^\circ + 360^\circ k \\ x = 270^\circ + 360^\circ k \end{cases} \quad x = 90^\circ + 180^\circ k$$

22) $\operatorname{sen} x + \sqrt{3} \cos x = 2$

$$\operatorname{sen} x + \sqrt{3} \cos x = 2 \quad \frac{1}{2} \operatorname{sen} x + \frac{\sqrt{3}}{2} \cos x = 1$$

$$\operatorname{sen}(x + 60^\circ) = 1 \quad x + 60^\circ = 90^\circ + 360^\circ k$$

$$x = 30^\circ + 360^\circ k$$

$$\begin{cases} 2x = 30^\circ + 360^\circ k & x = 15^\circ + 180^\circ k \\ 2x = 150^\circ + 360^\circ k & x = 75^\circ + 180^\circ k \end{cases}$$

24) $4\text{sen}(x - 30^\circ)\cos(x - 30^\circ) = \sqrt{3}$

$$2[2\text{sen}(x - 30^\circ)\cos(x - 30^\circ)] = \sqrt{3}$$

$$\text{sen}2(x - 30^\circ) = \frac{\sqrt{3}}{2}$$

$$2(x - 30^\circ) = 60^\circ + 360^\circ k \quad x = 60^\circ + 180^\circ k$$

$$2(x - 30^\circ) = 120^\circ + 360^\circ k \quad x = 90^\circ + 180^\circ k$$

25) $2\cos x = 3\text{tg} x$

$$2\cos x = \frac{3\text{sen} x}{\cos x} \quad 2\cos^2 x = 3\text{sen} x$$

$$2(1 - \text{sen}^2 x) = 3\text{sen} x \quad 2 - 2\text{sen}^2 x = 3\text{sen} x$$

26) $2\text{sen}^2 x + 3\text{sen} x - 2 = 0$

$$\text{sen} x = \frac{-3 \pm \sqrt{9 + 16}}{4} = \frac{-3 \pm 5}{4}$$

$$\text{sen} x = \frac{1}{2} \Rightarrow \begin{cases} x_1 = 30^\circ + 360^\circ k \\ x_2 = 150^\circ + 360^\circ k \end{cases}$$

$\text{sen} x = -2$ Sin solución porque $-1 \leq \text{sen} x \leq 1$

27) $\text{sen} 2x \cdot \cos x = 6\text{sen}^3 x$

$$2\text{sen} x \cdot \cos x \cdot \cos x = 6\text{sen}^3 x$$

$$\text{sen} x (\cos^2 x - 3\text{sen}^2 x) = 0$$

$$\text{sen} x = 0 \Rightarrow \begin{cases} x = 0^\circ + 360^\circ k \\ x = 180^\circ + 360^\circ k \end{cases} \Rightarrow x = 0^\circ + 180^\circ k$$

28) $\cos^2 x - 3\text{sen}^2 x = 0 \quad \cos^2 x = 3\text{sen}^2 x$

$$\text{tg}^2 x = \frac{1}{3} \quad \text{tg} x = \pm \frac{\sqrt{3}}{3}$$

$$\operatorname{tg} x = -\frac{\sqrt{3}}{3} \Rightarrow x = 150^\circ + 180^\circ k$$

$$\mathbf{29)} \quad 4 \operatorname{sen} \frac{x}{2} + 2 \cos x = 3$$

$$4 \operatorname{sen} \frac{x}{2} + 2 \left(\cos^2 \frac{x}{2} - \operatorname{sen}^2 \frac{x}{2} \right) = 3$$

$$4 \operatorname{sen} \frac{x}{2} + 2 \cos^2 \frac{x}{2} - 2 \operatorname{sen}^2 \frac{x}{2} = 3$$

$$4 \operatorname{sen} \frac{x}{2} + 2 \left(1 - \operatorname{sen}^2 \frac{x}{2} \right) - 2 \operatorname{sen}^2 \frac{x}{2} = 3$$

$$4 \operatorname{sen}^2 \frac{x}{2} - 4 \operatorname{sen} \frac{x}{2} + 1 = 0$$

$$\left(2 \operatorname{sen} \frac{x}{2} - 1 \right)^2 = 0 \qquad 2 \operatorname{sen} \frac{x}{2} - 1 = 0$$

$$\operatorname{sen} \frac{x}{2} = \frac{1}{2} \begin{cases} \frac{x}{2} = 30^\circ + 360^\circ k & x = 60^\circ + 360^\circ k \\ \frac{x}{2} = 150^\circ + 360^\circ k & x = 300^\circ + 360^\circ k \end{cases}$$

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