

Examen de Matemáticas I – 1º de Bachillerato

1. Sabiendo que $\cot \alpha = \frac{4}{3}$, hallar $\sin 2\alpha$, $\cos 2\alpha$ y $\operatorname{tg} 2\alpha$. **(2 puntos)**
2. Dos individuos A y B observan un globo que está situado en un plano vertical que pasa por ellos. La distancia entre los individuos es de 4 Km. Los ángulos de elevación del globo desde los observadores son 46° y 52° , respectivamente. Halla la altura del globo y su distancia a cada observador. **(2 puntos)**
3. Simplifica al máximo la expresión $\frac{\sin \alpha \cdot \cos \alpha \cdot (1 - \operatorname{tg}^2 \alpha)}{\cos^2 \alpha - \sin^2 \alpha}$. **(1 punto)**
4. Comprueba que la igualdad $\frac{\cos x + \operatorname{tg} x}{\cos x \cdot \operatorname{tg} x} = \operatorname{cotg} x + \sec x$, es cierta. **(1 punto)**
5. Resuelve la ecuación trigonométrica $\frac{\operatorname{cot} x + \operatorname{tg} x}{\operatorname{cot} x - \operatorname{tg} x} = 2$. **(2 puntos)**
6. Resuelve el sistema de ecuaciones trigonométricas $\begin{cases} \cos x + \cos y = 1 \\ x + y = 360^\circ \end{cases}$. **(2 puntos)**

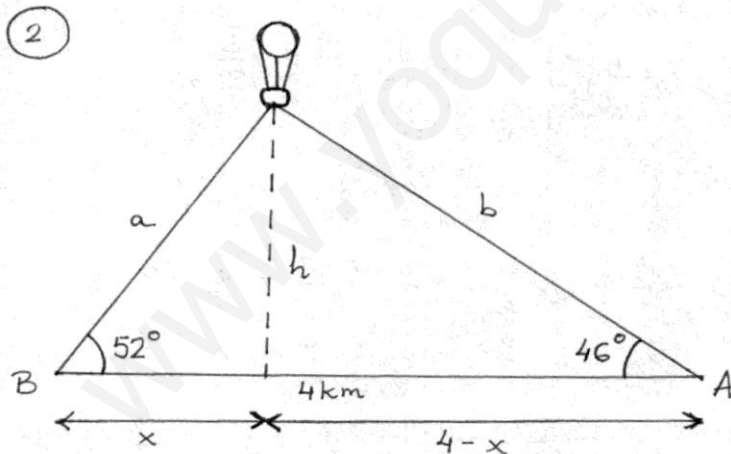
$$\begin{aligned} \textcircled{1} \quad \cotg \alpha &= \frac{4}{3} \Rightarrow \frac{\cos \alpha}{\sin \alpha} = \frac{4}{3} \Rightarrow \cos \alpha = \frac{4}{3} \sin \alpha \Rightarrow \\ &\Rightarrow \cos^2 \alpha = \frac{16}{9} \sin^2 \alpha. \text{ Como } \sin^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \\ &\Rightarrow \sin^2 \alpha + \frac{16}{9} \sin^2 \alpha = 1 \Rightarrow 9 \sin^2 \alpha + 16 \sin^2 \alpha = 9 \\ &\Rightarrow 25 \sin^2 \alpha = 9 \Rightarrow \sin^2 \alpha = \frac{9}{25} \Rightarrow \sin \alpha = \frac{3}{5} \\ &(\alpha \text{ está en el primer cuadrante}). \end{aligned}$$

$$\text{Entonces } \cos \alpha = \frac{4}{3} \cdot \frac{3}{5} \Rightarrow \cos \alpha = \frac{4}{5}$$

$$* \sin 2\alpha = 2 \sin \alpha \cos \alpha = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} \Rightarrow \sin 2\alpha = \frac{24}{25}$$

$$* \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} \\ \Rightarrow \cos 2\alpha = \frac{7}{25}$$

$$* \operatorname{tg} 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{24/25}{7/25} \Rightarrow \operatorname{tg} 2\alpha = \frac{24}{7}$$



$$\left. \begin{aligned} \operatorname{tg} 52 &= \frac{h}{x} \\ \operatorname{tg} 46 &= \frac{h}{4-x} \end{aligned} \right\} \Rightarrow$$

$$\left. \begin{aligned} h &= x \cdot \operatorname{tg} 52 \\ h &= (4-x) \operatorname{tg} 46 \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow x \operatorname{tg} 52 = (4-x) \operatorname{tg} 46 \Rightarrow x \operatorname{tg} 52 = 4 \operatorname{tg} 46 - x \operatorname{tg} 46$$

$$\Rightarrow x \operatorname{tg} 52 + x \operatorname{tg} 46 = 4 \operatorname{tg} 46 \Rightarrow x (\operatorname{tg} 52 + \operatorname{tg} 46) = 4 \operatorname{tg} 46$$

$$\Rightarrow x = \frac{4 \operatorname{tg} 46}{\operatorname{tg} 52 + \operatorname{tg} 46} \cong 1.79; \quad h = x \operatorname{tg} 52 = 1.79 \cdot \operatorname{tg} 52 \Rightarrow \underline{\underline{h \cong 2.29 \text{ km}}}$$

$$\sin 52^\circ = \frac{h}{a} \Rightarrow a = \frac{h}{\sin 52^\circ} = \frac{2.29}{\sin 52^\circ} \Rightarrow \underline{\underline{a \cong 2.91 \text{ km}}}$$

$$\sin 46^\circ = \frac{h}{b} \Rightarrow b = \frac{h}{\sin 46^\circ} = \frac{2.29}{\sin 46^\circ} \Rightarrow \underline{\underline{b \cong 3.18 \text{ km}}}$$

$$\begin{aligned} \textcircled{3} \quad \frac{\operatorname{sen} \alpha \cos \alpha (1 - \operatorname{tg}^2 \alpha)}{\cos^2 \alpha - \operatorname{sen}^2 \alpha} &= \frac{\operatorname{sen} \alpha \cos \alpha \left(1 - \frac{\operatorname{sen}^2 \alpha}{\cos^2 \alpha}\right)}{\cos^2 \alpha - \operatorname{sen}^2 \alpha} = \\ &= \frac{\operatorname{sen} \alpha \cos \alpha \frac{\cos^2 \alpha - \operatorname{sen}^2 \alpha}{\cos^2 \alpha}}{\cos^2 \alpha - \operatorname{sen}^2 \alpha} = \frac{\operatorname{sen} \alpha \cos \alpha (\cos^2 \alpha - \operatorname{sen}^2 \alpha)}{\cos^2 \alpha (\cos^2 \alpha - \operatorname{sen}^2 \alpha)} = \\ &= \frac{\operatorname{sen} \alpha}{\cos \alpha} = \underline{\underline{\operatorname{tg} \alpha}} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad \frac{\cos x + \operatorname{tg} x}{\cos x \cdot \operatorname{tg} x} &= \frac{\cos x + \frac{\operatorname{sen} x}{\cos x}}{\cos x \cdot \frac{\operatorname{sen} x}{\cos x}} = \frac{\frac{\cos^2 x + \operatorname{sen} x}{\cos x}}{\operatorname{sen} x} = \\ &= \frac{\cos^2 x + \operatorname{sen} x}{\cos x \cdot \operatorname{sen} x} = \frac{\cos^2 x}{\cos x \cdot \operatorname{sen} x} + \frac{\operatorname{sen} x}{\cos x \cdot \operatorname{sen} x} = \\ &= \frac{\cos x}{\operatorname{sen} x} + \frac{1}{\cos x} = \underline{\underline{\operatorname{cotg} x + \operatorname{sec} x}} \end{aligned}$$

$$\textcircled{5} \quad \frac{\operatorname{cotg} x + \operatorname{tg} x}{\operatorname{cotg} x - \operatorname{tg} x} = 2 \Rightarrow \frac{\frac{1}{\operatorname{tg} x} + \operatorname{tg} x}{\frac{1}{\operatorname{tg} x} - \operatorname{tg} x} = 2 \Rightarrow \frac{\frac{1 + \operatorname{tg}^2 x}{\operatorname{tg} x}}{\frac{1 - \operatorname{tg}^2 x}{\operatorname{tg} x}} = 2$$

$$\Rightarrow \frac{1 + \operatorname{tg}^2 x}{1 - \operatorname{tg}^2 x} = 2 \Rightarrow 1 + \operatorname{tg}^2 x = 2 - 2\operatorname{tg}^2 x \Rightarrow 3\operatorname{tg}^2 x = 1$$

$$\Rightarrow \operatorname{tg}^2 x = \frac{1}{3} \Rightarrow \operatorname{tg} x = \pm \sqrt{\frac{1}{3}} = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

$$x_1 = 30^\circ + 360^\circ k$$

$$x_3 = 150^\circ + 360^\circ k$$

$$x_2 = 210^\circ + 360^\circ k$$

$$x_4 = 330^\circ + 360^\circ k$$

$$\textcircled{6} \quad \cos x + \cos y = 1 \Rightarrow 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} = 1 \Rightarrow$$

$$2 \cos 180^\circ \cos \frac{x-y}{2} = 1 \Rightarrow -2 \cos \frac{x-y}{2} = 1 \Rightarrow$$

$$\cos \frac{x-y}{2} = -\frac{1}{2} \Rightarrow \frac{x-y}{2} = 120 \quad (1) \quad \vee \quad \frac{x-y}{2} = 240 \quad (2)$$

$$\text{En el caso (1)} \quad \left. \begin{array}{l} x-y = 240 \\ x+y = 360 \end{array} \right\} \Rightarrow \boxed{\begin{array}{l} x_1 = 300^\circ + 360^\circ k \\ y_1 = 60^\circ + 360^\circ k \end{array}}$$

$$\text{En el caso (2)} \quad \left. \begin{array}{l} x-y = 480 \\ x+y = 360 \end{array} \right\} \Rightarrow \boxed{\begin{array}{l} x_2 = 420^\circ + 360^\circ k \\ y_2 = -60^\circ + 360^\circ k \end{array}} \Rightarrow$$

$$\text{soluciones equivalentes a } \boxed{\begin{array}{l} x_2 = 60^\circ + 360^\circ k \\ y_2 = 300^\circ + 360^\circ k \end{array}}$$