

EXAMEN DE TRIGONOMETRÍA

1. Resuelve:

$$\cos^2 \frac{x}{2} + \cos x - \frac{1}{2} = 0$$

2. Demuestra la siguiente identidad trigonométrica:

$$\operatorname{sen} 2\alpha \cdot \cos \alpha - \operatorname{sen} \alpha \cdot \cos 2\alpha = \operatorname{sen} \alpha$$

3. Si $\cos \alpha = -\frac{1}{4}$ y $\alpha < \pi$, halla:

a) $\operatorname{sen} \alpha$ b) $\cos\left(\frac{\pi}{3} + \alpha\right)$ c) $\operatorname{tg} \frac{\alpha}{2}$ d) $\operatorname{sen}\left(\frac{\pi}{6} - \alpha\right)$

4. Desde un punto del suelo medimos el ángulo bajo el que se ve un edificio y obtenemos 40° . Nos alejamos 30 m y el ángulo es ahora de 28° . Calcula la altura del edificio y la distancia desde la que se hizo la primera observación.

$$\textcircled{1} \cos^2 \frac{x}{2} + \cos x - \frac{1}{2} = 0 \quad \text{Como } \cos \frac{x}{2} = \pm \sqrt{\frac{1+\cos x}{2}}$$

$$\left(\sqrt{\frac{1+\cos x}{2}} \right)^2 + \cos x - \frac{1}{2} = 0; \quad \frac{1+\cos x}{2} + \frac{2\cos x}{2} - \frac{1}{2} = 0;$$

$$3\cos x = 0; \quad \cos x = 0; \quad x = \text{Arccos}(0) = \begin{matrix} 90^\circ \\ 270^\circ \end{matrix} + 360^\circ k, \quad k \in \mathbb{Z}$$

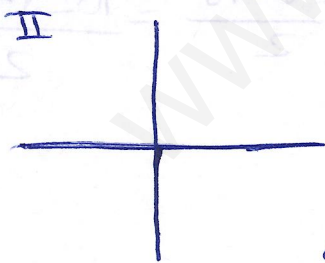
$$\textcircled{2} \sin 2\alpha \cdot \cos \alpha - \sin \alpha \cdot \cos 2\alpha = \sin \alpha.$$

Como $\begin{cases} \sin 2\alpha = 2\sin \alpha \cos \alpha \\ \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \end{cases}$ } Substituímos en el primer miembro de la identidad

$$\begin{aligned} 2\sin \alpha \cos \alpha \cdot \cos \alpha - \sin \alpha (\cos^2 \alpha - \sin^2 \alpha) &= 2\sin \alpha \cos^2 \alpha - \sin \alpha \cos^2 \alpha \\ + \sin^3 \alpha &= \sin \alpha \cos^2 \alpha + \sin^3 \alpha = \sin \alpha \cdot (\underbrace{\cos^2 \alpha + \sin^2 \alpha}_{1}) = \\ &= \sin \alpha. \end{aligned}$$

(Como queríamos demostrar)

$$\textcircled{3} \cos \alpha = -\frac{1}{4} \text{ y } \alpha < \pi \Rightarrow \text{Estamos en el 2º cuadrante.}$$



$$a) \sin^2 \alpha + \cos^2 \alpha = 1; \quad \sin^2 \alpha = 1 - \cos^2 \alpha;$$

$$\sin^2 \alpha = 1 - \left(-\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\sin \alpha = \sqrt{\frac{15}{16}}; \quad \boxed{\sin \alpha = \frac{\sqrt{15}}{4}}$$

$$\begin{aligned} b) \cos\left(\frac{\pi}{3} + \alpha\right) &= \cos \frac{\pi}{3} \cdot \cos \alpha - \sin \frac{\pi}{3} \cdot \sin \alpha = \frac{1}{2} \cdot \left(-\frac{1}{4}\right) - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{15}}{4} = \\ &= -\frac{1}{8} - \frac{\sqrt{45}}{8} = \frac{-1 - \sqrt{45}}{8} = \frac{-1 - 3\sqrt{5}}{8} \end{aligned}$$

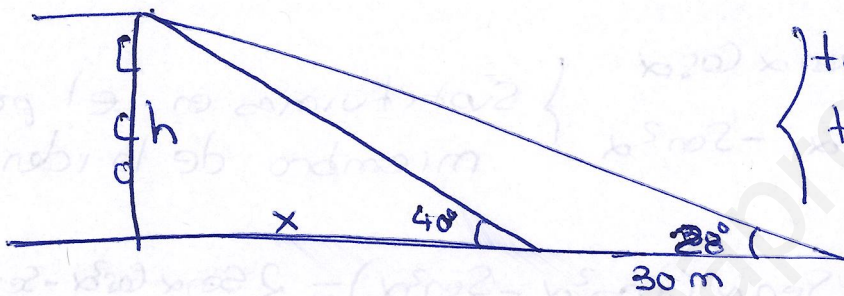
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$$c) \operatorname{tg} \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} \rightarrow \text{Si } \alpha < \pi \Rightarrow \frac{\alpha}{2} < \frac{\pi}{2} \Rightarrow \frac{\alpha}{2} \text{ está en el 1}^{\text{er}} \text{ Cuadrante} \Rightarrow \operatorname{tg} \alpha > 0$$

$$\left[\operatorname{tg} \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \frac{1}{4}}{1 - \frac{1}{4}}} = \pm \sqrt{\frac{5/4}{3/4}} = \pm \sqrt{\frac{5}{3}} = \frac{\sqrt{15}}{3} \right]$$

$$d) \left[\begin{aligned} \operatorname{Sen} \left(\frac{\pi}{6} - \alpha \right) &= \operatorname{Sen} \frac{\pi}{6} \cos \alpha + \cos \frac{\pi}{6} \cdot \operatorname{Sen} \alpha = \\ &= \frac{1}{2} \cdot \left(-\frac{1}{4} \right) + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{15}}{4} = -\frac{1}{8} + \frac{\sqrt{45}}{8} = \frac{-1 + 3\sqrt{15}}{8} \end{aligned} \right]$$

4)



$$\left. \begin{aligned} \operatorname{tg} 40^\circ &= \frac{h}{x} \\ \operatorname{tg} 28^\circ &= \frac{h}{x+30} \end{aligned} \right\} \rightarrow h = x \cdot \operatorname{tg} 40^\circ$$

$$\operatorname{tg} 28^\circ = \frac{x \cdot \operatorname{tg} 40^\circ}{x+30} ; \operatorname{tg} 28^\circ (x+30) = x \operatorname{tg} 40^\circ ; x \operatorname{tg} 28^\circ + 30 \operatorname{tg} 28^\circ = x \operatorname{tg} 40^\circ$$

$$0'53171x + 15'951 = 0'8391x$$

$$\boxed{x = 51'892 \text{ m}}$$

$$\boxed{h = 51'892 \cdot \operatorname{tg} 40^\circ = 43'543 \text{ m}}$$