

Ejercicios de Identidades y ecuaciones trigonométricas

1. Demuestra si las siguientes identidades trigonométricas son ciertas:

a. $\frac{2\operatorname{tg}x}{\operatorname{sen}2x} = 1 + \operatorname{tg}^2x$

b. $1 - 2\cos 2x = 4\operatorname{sen}^2x - 1$

c. $\frac{1 + \cos x}{\operatorname{sen} x} = \frac{\operatorname{sen} x}{1 - \cos x}$

d. $\operatorname{tg} \alpha + \operatorname{tg} \beta = \frac{\operatorname{sen}(\alpha + \beta)}{\cos \alpha \cdot \cos \beta}$

e. $\sec^2x - \cos^2x = \operatorname{tg}^2x + \operatorname{sen}^2x$

f. $\frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{\operatorname{sen}(\alpha + \beta) + \operatorname{sen}(\alpha - \beta)} = \operatorname{tg} \beta$

k. $\frac{\cos \theta \cdot \operatorname{sen}(2\theta + x) - \operatorname{sen} 2\theta \cdot \cos(\theta + x)}{\cos \theta} = \operatorname{sen} x$

g. $\frac{1}{\cos \alpha(1 + \cos \alpha)} = \frac{\operatorname{tg} \alpha - \operatorname{sen} \alpha}{\operatorname{sen}^3 \alpha}$

h. $\operatorname{tg}(-x) = -\operatorname{tg} x$

i. $\frac{\operatorname{tga} - \operatorname{cotga}}{\operatorname{tga} + \operatorname{cotga}} = 2\operatorname{sen}^2a - 1$

j. $\frac{\operatorname{sen}^2a}{\cos a} + \frac{2\operatorname{sen} a}{\operatorname{tg}2a} = \cos a$

2. Siendo $\alpha + \beta + \delta = 180^\circ$ demuestra que:

a. $\operatorname{tg}(\alpha + \beta) + \operatorname{tg} \delta = 0$

b. $\operatorname{sen}(\alpha + \beta) = \operatorname{sen} \delta$

3. Demuestra que si $a + b = \pi$, entonces:

a. $\frac{(\operatorname{sen} a + \operatorname{sen} b)(\cos a - \cos b)}{\operatorname{sen}2b} = -2$

4. Resuelve las siguientes ecuaciones y sistemas

a. $\cos x + \operatorname{sen} x = 1$

b. $\operatorname{sen} 3x = \frac{\sqrt{2}}{2}$

c. $2\cos x = 3\operatorname{tg}x$

d. $\operatorname{sen} 2x \cdot \cos x = 6\operatorname{sen}^3x$

e. $\operatorname{sen} x - \operatorname{sen} 2x = 0$

f. $\cos x - \operatorname{sen} 2x = 0$

g. $2\cos x = 3(1 - \operatorname{sen} x)$

h. $\left. \begin{aligned} x + y &= \frac{\pi}{2} \\ \operatorname{sen} x + \operatorname{sen} y &= \sqrt{2} \end{aligned} \right\}$

i. $\left. \begin{aligned} x - y &= \frac{\pi}{6} \\ \operatorname{sen} x \cdot \operatorname{sen} y &= \cos x \cdot \cos y \end{aligned} \right\}$

j. $\left. \begin{aligned} \operatorname{sen} x \cdot \cos y &= \frac{3}{4} \\ \cos x \cdot \operatorname{sen} y &= \frac{1}{4} \end{aligned} \right\}$