

## EJERCICIOS RESUELTOS DE ECUACIONES TRIGONOMÉTRICAS

1.-

Resuelve las siguientes ecuaciones y da los resultados en grados y en radianes.

a)  $\operatorname{sen} x = 1$

c)  $2 \cos x + 1 = 0$

b)  $\operatorname{tg} x = 0$

d)  $\sqrt{3} \operatorname{tg} x - 1 = 0$

2.-

(TIC) Resuelve las siguientes ecuaciones trigonométricas indicando todas sus soluciones en grados.

a)  $\operatorname{sen} x = \frac{1}{2}$

c)  $\operatorname{tg} x = 1$

e)  $\cos x = \frac{1}{2}$

g)  $\operatorname{sen} x = 0$

b)  $\cos x = \frac{\sqrt{3}}{2}$

d)  $\operatorname{sen} x = -\frac{\sqrt{2}}{2}$

f)  $\operatorname{tg} x = -\frac{\sqrt{3}}{3}$

h)  $1 + \cos x = 0$

3.-

(TIC) Resuelve las siguientes ecuaciones trigonométricas indicando todas sus soluciones en radianes.

a)  $\operatorname{sen} 4x = -\frac{\sqrt{3}}{2}$

c)  $\operatorname{tg} 3x = -1$

e)  $\cos \frac{x}{3} = -\frac{1}{2}$

b)  $\cos 2x = \frac{\sqrt{2}}{2}$

d)  $\operatorname{sen} \frac{x}{2} = 0$

f)  $\operatorname{tg} \frac{3x}{4} = -\frac{\sqrt{3}}{3}$

4.-

(TIC) Halla todas las soluciones de las siguientes ecuaciones trigonométricas.

a)  $\operatorname{sen} x = \cos x$

b)  $\operatorname{sen} 2x - \operatorname{sen} x = 0$

c)  $\operatorname{sen} x - \sqrt{3} \cos x = 0$

d)  $\operatorname{sen} x + \cos x = \sqrt{2}$

5.-

(TIC) Resuelve las siguientes ecuaciones trigonométricas en el intervalo  $[0^\circ, 360^\circ]$ .

a)  $\operatorname{tg} x + 4 \operatorname{cotg} x = 5$

b)  $8 \cos 2x = 8 \cos x - 9$

c)  $\operatorname{tg} 2x = \operatorname{cotg} x$

d)  $2 \operatorname{sen}^2 x + \cos 2x = 4 \cos^2 x$

6.-

(TIC) Halla las soluciones de las siguientes ecuaciones trigonométricas comprendidas en el intervalo  $[0, 2\pi]$ .

a)  $\operatorname{sen}^2 x + \operatorname{tg}^2 x = 0$

b)  $2 \operatorname{sen} x + \sqrt{3} \operatorname{tg} x = 0$

c)  $\cos 2x - \operatorname{sen} x - \operatorname{sen} 2x - \cos x$

7.-

(TIC) Resuelve las siguientes ecuaciones trigonométricas en el intervalo  $[-\pi, \pi]$ .

a)  $\operatorname{sen} 3x + \operatorname{sen} 6x = 0$

b)  $\cos 5x + \cos 3x = \cos x$

c)  $\sqrt{3} \cos x + \operatorname{sen} x = 2$

8.-

Resuelve la ecuación trigonométrica  $\operatorname{sen}^4 x - \cos^4 x = \frac{1}{2}$

9.-

Resuelve las siguientes ecuaciones.

a)  $\cos x \operatorname{tg} x = \frac{1}{2}$

f)  $\operatorname{tg} x + \operatorname{sen} x = 0$

b)  $\cos 2x + \operatorname{sen} 2x = 1$

g)  $\operatorname{tg} x - \operatorname{sen} 2x = 0$

c)  $\cos 2x - \operatorname{sen} 2x = 0$

h)  $\frac{\operatorname{sen}(60^\circ - x)}{\cos x} = 1$

d)  $\operatorname{sen} 2x + \cos x = 1$

i)  $\operatorname{tg}\left(\frac{\pi}{4} - x\right) + \operatorname{tg} x - 1 = 0$

e)  $\operatorname{sen} 2x + \operatorname{sen} 2x = 0$

j)  $\operatorname{sen}(x + 30^\circ) + \cos(x + 60^\circ) = 1 + \cos 2x$

10.-

a)  $4 \operatorname{sen} x - \sec x = 0$

b)  $\frac{\cos^2 x}{2 \cos x + \operatorname{sen} x} = \operatorname{sen} x$

c)  $\frac{1}{\cos x + \operatorname{sen} x} + 2 \operatorname{sen} x = 2 \cos x$

d)  $\operatorname{sen} x (\operatorname{sen} x - 1) = 5 \cos^2 x - 4$

e)  $2 \cos x - 1 = \sec x$

f)  $2 \cos x + \operatorname{sen} x = 1$

g)  $\operatorname{sen} x + \cos x = 0$

## SOLUCIONES

1.-

a)  $\operatorname{sen} x = 1$  El seno de un ángulo vale 1 únicamente en  $90^\circ, 450^\circ, 810^\circ, \text{etc.}$ Por tanto:  $x = 90^\circ + 360^\circ k$  con  $k \in \mathbb{Z}$  o  $x = \frac{\pi}{2} + 2\pi k$  con  $k \in \mathbb{Z}$ b)  $\operatorname{tg} x = 0$  La tangente vale 0 en los ángulos  $0^\circ, 180^\circ, 360^\circ, 540^\circ, \text{etc.}$ Por tanto:  $x = 180^\circ k$  con  $k \in \mathbb{Z}$  o  $x = \pi k$  con  $k \in \mathbb{Z}$ c)  $\cos x = -\frac{1}{2}$  El coseno es negativo para los ángulos de los cuadrantes 2.º y 3.ºPor tanto:  $x = 120^\circ + 360^\circ k, x = 240^\circ + 360^\circ k$  con  $k \in \mathbb{Z}$  o  $x = \frac{2\pi}{3} + 2\pi k, x = \frac{4\pi}{3} + 2\pi k$  con  $k \in \mathbb{Z}$ d)  $\operatorname{tg} x = \frac{\sqrt{3}}{3}$  La tangente es positiva para los ángulos de los cuadrantes 1.º y 3.ºPor tanto:  $x = 30^\circ + 360^\circ k, x = 210^\circ + 360^\circ k$  con  $k \in \mathbb{Z}$  o  $x = \frac{\pi}{6} + 2\pi k, x = \frac{7\pi}{6} + 2\pi k$  con  $k \in \mathbb{Z}$ 

2.-

a)  $\operatorname{sen} x = \frac{1}{2} \Rightarrow \begin{cases} x = 30^\circ + 360^\circ k \\ x = 150^\circ + 360^\circ k \end{cases}$  d)  $\operatorname{sen} x = -\frac{\sqrt{2}}{2} \Rightarrow \begin{cases} x = 225^\circ + 360^\circ k \\ x = 315^\circ + 360^\circ k \end{cases}$  f)  $\operatorname{sen} x = 0 \Rightarrow x = 180^\circ k$ b)  $\cos x = \frac{\sqrt{3}}{2} \Rightarrow \begin{cases} x = 30^\circ + 360^\circ k \\ x = 330^\circ + 360^\circ k \end{cases}$  e)  $\cos x = -\frac{1}{2} \Rightarrow \begin{cases} x = 120^\circ + 360^\circ k \\ x = 240^\circ + 360^\circ k \end{cases}$  g)  $1 - \cos x = 0 \Rightarrow x = 360^\circ k$ c)  $\operatorname{tg} x = 1 \Rightarrow \begin{cases} x = 45^\circ + 360^\circ k \\ x = 225^\circ + 360^\circ k \end{cases}$  f)  $\operatorname{tg} x = -\frac{\sqrt{3}}{2} \Rightarrow \begin{cases} x = 150^\circ + 360^\circ k \\ x = 330^\circ + 360^\circ k \end{cases}$ 

3.-

a)  $\operatorname{sen} 4x = -\frac{\sqrt{3}}{2} \Rightarrow \begin{cases} 4x = \frac{4\pi}{3} + 2\pi k \\ 4x = \frac{5\pi}{3} + 2\pi k \end{cases} \Rightarrow \begin{cases} x = \frac{\pi}{3} + \frac{\pi k}{2} \\ x = \frac{5\pi}{12} + \frac{\pi k}{2} \end{cases}$  d)  $\operatorname{sen} \frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \pi k \Rightarrow x = 2\pi k$ b)  $\cos 2x = \frac{\sqrt{2}}{2} \Rightarrow \begin{cases} 2x = \frac{\pi}{4} + 2\pi k \\ 2x = \frac{7\pi}{4} + 2\pi k \end{cases} \Rightarrow \begin{cases} x = \frac{\pi}{8} + \pi k \\ x = \frac{7\pi}{8} + \pi k \end{cases}$  e)  $\cos \frac{x}{3} = -\frac{1}{2} \Rightarrow \begin{cases} \frac{x}{3} = \frac{2\pi}{3} + 2\pi k \\ \frac{x}{3} = \frac{4\pi}{3} + 2\pi k \end{cases} \Rightarrow \begin{cases} x = 2\pi + 6\pi k \\ x = 4\pi + 6\pi k \end{cases}$ c)  $\operatorname{tg} 3x = -1 \Rightarrow \begin{cases} 3x = \frac{3\pi}{4} + 2\pi k \\ 3x = \frac{7\pi}{4} + 2\pi k \end{cases} \Rightarrow \begin{cases} x = \frac{\pi}{4} + \frac{2\pi k}{3} \\ x = \frac{7\pi}{12} + \frac{2\pi k}{3} \end{cases}$  f)  $\operatorname{tg} \frac{3x}{4} = -\frac{\sqrt{3}}{3} \Rightarrow \begin{cases} \frac{3x}{4} = \frac{5\pi}{6} + 2\pi k \\ \frac{3x}{4} = \frac{11\pi}{6} + 2\pi k \end{cases} \Rightarrow \begin{cases} x = \frac{10\pi}{9} + \frac{8\pi k}{3} \\ x = \frac{22\pi}{9} + \frac{8\pi k}{3} \end{cases}$ 

4.-

a)  $\operatorname{sen} x = \cos x \Rightarrow \operatorname{tg} x = 1 \Rightarrow \begin{cases} x = 45^\circ + 360^\circ k \\ x = 225^\circ + 360^\circ k \end{cases}$ b)  $\operatorname{sen} 2x - \operatorname{sen} x = 0 \Rightarrow 2 \operatorname{sen} x \cos x - \operatorname{sen} x = 0 \Rightarrow \operatorname{sen} x (2 \cos x - 1) = 0 \Rightarrow \begin{cases} \operatorname{sen} x = 0 \Rightarrow x = 180^\circ k \\ \cos x = \frac{1}{2} \Rightarrow \begin{cases} x = 60^\circ + 360^\circ k \\ x = 300^\circ + 360^\circ k \end{cases} \end{cases}$ c)  $\operatorname{sen} x - \sqrt{3} \cos x = 0 \Rightarrow \operatorname{tg} x = \sqrt{3} \Rightarrow \begin{cases} x = 60^\circ + 360^\circ k \\ x = 240^\circ + 360^\circ k \end{cases}$ d)  $\operatorname{sen} x + \cos x = \sqrt{2} \Rightarrow \operatorname{sen} x + \sqrt{1 - \operatorname{sen}^2 x} = \sqrt{2} \Rightarrow 1 - \operatorname{sen}^2 x = 2 + \operatorname{sen}^2 x - 2\sqrt{2} \operatorname{sen} x \Rightarrow$   
 $\Rightarrow 2 \operatorname{sen}^2 x - 2\sqrt{2} \operatorname{sen} x + 1 = 0 \Rightarrow \operatorname{sen} x = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} \Rightarrow x = 45^\circ + 360^\circ k$

5.-

$$a) \operatorname{tg} x + 4 \cot g x = 5 \Rightarrow \operatorname{tg} x + \frac{4}{\operatorname{tg} x} = 5 \Rightarrow \operatorname{tg}^2 x + 4 = 5 \operatorname{tg} x \Rightarrow \operatorname{tg}^2 x - 5 \operatorname{tg} x + 4 = 0 \Rightarrow$$

$$\operatorname{tg} x = \frac{5 \pm \sqrt{25 - 16}}{2} \Rightarrow \begin{cases} \operatorname{tg} x = 4 \Rightarrow x = 75^\circ 58' \quad x = 255^\circ 58' \\ \operatorname{tg} x = 1 \Rightarrow x = 45^\circ \quad x = 225^\circ \end{cases}$$

$$b) 8 \cos 2x = 8 \cos x - 9 \Rightarrow 8 \cos^2 x - 8 \operatorname{sen}^2 x - 8 \cos x + 9 = 0 \Rightarrow$$

$$\Rightarrow 8 \cos^2 x - 8 + 8 \cos^2 x - 8 \cos x + 9 = 0 \Rightarrow 16 \cos^2 x - 8 \cos x + 1 = 0 \Rightarrow$$

$$\Rightarrow \cos x = \frac{8 \pm \sqrt{64 - 64}}{32} = \frac{1}{4} \Rightarrow x = 75^\circ 31' ; x = 284^\circ 29'$$

$$c) \operatorname{tg} 2x = \cot g x \Rightarrow \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} = \frac{1}{\operatorname{tg} x} \Rightarrow \frac{2 \operatorname{tg}^2 x}{1 - \operatorname{tg}^2 x} = 1 \Rightarrow 2 \operatorname{tg}^2 x = 1 - \operatorname{tg}^2 x \Rightarrow \operatorname{tg}^2 x = \frac{1}{3} \Rightarrow \operatorname{tg} x = \pm \frac{\sqrt{3}}{3} \Rightarrow \begin{cases} x = 30^\circ, x = 210^\circ \\ x = 150^\circ, x = 330^\circ \end{cases}$$

$$d) 2 \operatorname{sen}^2 x + \cos 2x = 4 \cos^2 x \Rightarrow 2 \operatorname{sen}^2 x + \cos^2 x - \operatorname{sen}^2 x = 4 \cos^2 x \Rightarrow \operatorname{sen}^2 x + \cos^2 x = 4 \cos^2 x \Rightarrow 1 = 4 \cos^2 x \Rightarrow$$

$$\Rightarrow \cos^2 x = \frac{1}{4} \Rightarrow \cos x = \pm \frac{1}{2} \Rightarrow \begin{cases} x = 60^\circ, x = 300^\circ \\ x = 120^\circ, x = 240^\circ \end{cases}$$

6.-

$$a) \operatorname{sen}^2 x + \operatorname{tg}^2 x = 0 \Rightarrow \operatorname{sen}^2 x \left( 1 + \frac{1}{\cos^2 x} \right) = 0 \Rightarrow \begin{cases} \operatorname{sen}^2 x = 0 \Rightarrow \operatorname{sen} x = 0 \Rightarrow x = 0, x = \pi, x = 2\pi \\ 1 + \frac{1}{\cos^2 x} = 0 \text{ no aporta soluciones} \end{cases}$$

$$b) 2 \operatorname{sen} x + \sqrt{3} \cdot \operatorname{tg} x = 0 \Rightarrow 2 \operatorname{sen} x + \sqrt{3} \cdot \frac{\operatorname{sen} x}{\cos x} = 0 \Rightarrow \operatorname{sen} x \left( 2 + \frac{\sqrt{3}}{\cos x} \right) = 0 \Rightarrow \begin{cases} \operatorname{sen} x = 0 \Rightarrow x = 0, x = \pi, x = 2\pi \\ \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \frac{5\pi}{6}, x = \frac{7\pi}{6} \end{cases}$$

$$c) \cos 2x - \operatorname{sen} x = \operatorname{sen} 2x - \cos x \Rightarrow \cos 2x + \cos x = \operatorname{sen} 2x + \operatorname{sen} x \Rightarrow 2 \cos \frac{3x}{2} \cos \frac{x}{2} = 2 \operatorname{sen} \frac{3x}{2} \cos \frac{x}{2} \Rightarrow$$

$$\Rightarrow \cos \frac{x}{2} \left[ \cos \frac{3x}{2} - \operatorname{sen} \frac{3x}{2} \right] = 0 \Rightarrow \begin{cases} \cos \frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} ; \frac{x}{2} = \frac{3\pi}{2} \Rightarrow x = \pi, x = 3\pi \\ \operatorname{tg} \frac{3x}{2} = 1 \Rightarrow \frac{3x}{2} = \frac{\pi}{4} ; \frac{3x}{2} = \frac{5\pi}{4} \Rightarrow x = \frac{\pi}{6}, x = \frac{5\pi}{6} \end{cases}$$

7.-

$$a) \operatorname{sen} 3x + \operatorname{sen} 6x = 0 \Rightarrow 2 \operatorname{sen} \frac{9x}{2} \cos \frac{3x}{2} = 0 \Rightarrow \begin{cases} \operatorname{sen} \frac{9x}{2} = 0 \Rightarrow x = 0, x = -\frac{2\pi}{9}, x = \frac{2\pi}{9} \\ \cos \frac{3x}{2} = 0 \Rightarrow x = \pi, x = -\frac{\pi}{3}, x = \frac{\pi}{3} \end{cases}$$

$$b) \cos 5x + \cos 3x = \cos x \Rightarrow 2 \cos 4x \cos x = \cos x \Rightarrow \cos x (2 \cos 4x - 1) = 0 \Rightarrow$$

$$\Rightarrow \begin{cases} \cos x = 0 \Rightarrow x = \frac{\pi}{2}, x = -\frac{\pi}{2} \\ \cos 4x = \frac{1}{2} \Rightarrow x = \frac{5\pi}{12}, x = \frac{\pi}{12}, x = \frac{\pi}{12} \end{cases}$$

$$c) \sqrt{3} \cos x + \operatorname{sen} x = 2 \Rightarrow \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \operatorname{sen} x = 1 \Rightarrow \operatorname{sen} \left( x + \frac{\pi}{3} \right) = 1 \Rightarrow x = \frac{\pi}{6}$$

8.-

$$\operatorname{sen}^4 x - \cos^4 x = \frac{1}{2} \Rightarrow (\operatorname{sen}^2 x - \cos^2 x) (\operatorname{sen}^2 x + \cos^2 x) = \frac{1}{2} \Rightarrow \operatorname{sen}^2 x - \cos^2 x = \frac{1}{2} \Rightarrow -\cos 2x = \frac{1}{2} \Rightarrow \cos 2x = -\frac{1}{2}$$

$$\text{Soluciones: } 2x = 120^\circ + 360^\circ k \Rightarrow x = 60^\circ + 180^\circ k$$

$$2x = 240^\circ + 360^\circ k \Rightarrow x = 120^\circ + 180^\circ k$$

9.-

$$a) \cos x \operatorname{tg} x = \frac{1}{2} \rightarrow \operatorname{sen} x = \frac{1}{2} \rightarrow \begin{cases} x_1 = 30^\circ + 360^\circ \cdot k \\ x_2 = 150^\circ + 360^\circ \cdot k \end{cases}$$

$$b) \cos 2x + \operatorname{sen} 2x = 1 \rightarrow \cos^2 x - \operatorname{sen}^2 x + 2 \operatorname{sen} x \cdot \cos x = \cos^2 x + \operatorname{sen}^2 x \\ \rightarrow -2 \operatorname{sen}^2 x + 2 \operatorname{sen} x \cdot \cos x = 0 \rightarrow 2 \operatorname{sen} x (-\operatorname{sen} x + \cos x) = 0$$

$$\operatorname{sen} x = 0 \rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ \cdot k \\ x_2 = 180^\circ + 360^\circ \cdot k \end{cases}$$

$$\operatorname{sen} x = \cos x \rightarrow \begin{cases} x_1 = 45^\circ + 360^\circ \cdot k \\ x_2 = 225^\circ + 360^\circ \cdot k \end{cases}$$

$$c) \cos 2x - \operatorname{sen} 2x = 0 \rightarrow \cos 2x = \operatorname{sen} 2x \rightarrow \begin{cases} x_1 = 22,5^\circ + 180^\circ \cdot k \\ x_2 = 112,5^\circ + 180^\circ \cdot k \end{cases}$$

$$d) \operatorname{sen} 2x + \cos x = (2 \operatorname{sen} x + 1) \cos x = 0 \rightarrow \begin{cases} x_1 = 90^\circ + 360^\circ \cdot k & x_3 = 210^\circ + k \cdot 360^\circ \\ x_2 = 270^\circ + 360^\circ \cdot k & x_4 = 330^\circ + k \cdot 360^\circ \end{cases}$$

$$e) \operatorname{sen} 2x + \operatorname{sen} 2x = 0 \rightarrow 2 \operatorname{sen} 2x = 0 \rightarrow \begin{cases} x_1 = 0^\circ + 180^\circ \cdot k \\ x_2 = 90^\circ + 180^\circ \cdot k \end{cases}$$

$$f) \operatorname{tg} x + \operatorname{sen} x = 0 \rightarrow \operatorname{sen} x \left( \frac{1}{\cos x} + 1 \right) = 0$$

$$\operatorname{sen} x = 0 \rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ \cdot k \\ x_2 = 180^\circ + 360^\circ \cdot k \end{cases}$$

$$\frac{1}{\cos x} + 1 = 0 \rightarrow x_3 = 180^\circ + 360^\circ \cdot k$$

$$g) \operatorname{tg} x - \operatorname{sen} 2x = 0 \rightarrow \frac{\operatorname{sen} x}{\cos x} - 2 \operatorname{sen} x \cdot \cos x = 0 \rightarrow \operatorname{sen} x (1 - 2 \cos^2 x) = 0$$

$$\operatorname{sen} x = 0 \rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ \cdot k \\ x_2 = 180^\circ + 360^\circ \cdot k \end{cases}$$

$$1 - 2 \cos^2 x = 0 \rightarrow \cos x = \sqrt{\frac{1}{2}} \rightarrow x_3 = 45^\circ + 360^\circ \cdot k$$

$$h) \frac{\operatorname{sen} (60^\circ - x)}{\cos x} = 1 \rightarrow \frac{\sqrt{3} \cos x - \operatorname{sen} x}{2 \cos x} = 1 \rightarrow \sqrt{3} - \operatorname{tg} x = 2$$

$$\rightarrow \operatorname{tg} x = -0,2679 \rightarrow x = 345^\circ + 360^\circ \cdot k$$

$$i) \operatorname{tg} \left( \frac{\pi}{4} - x \right) + \operatorname{tg} x - 1 = 0 \rightarrow \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} + \operatorname{tg} x - 1 = 0$$

$$\rightarrow \operatorname{tg}^2 x - \operatorname{tg} x = 0 \rightarrow \operatorname{tg} x (\operatorname{tg} x - 1) = 0$$

$$\operatorname{tg} x = 0 \rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ \cdot k \\ x_2 = 180^\circ + 360^\circ \cdot k \end{cases}$$

$$\operatorname{tg} x - 1 = 0 \rightarrow \operatorname{tg} x = 1 \rightarrow \begin{cases} x_1 = 45^\circ + 360^\circ \cdot k \\ x_2 = 225^\circ + 360^\circ \cdot k \end{cases}$$

$$j) \quad \sin(x + 30^\circ) + \cos(x + 60^\circ) = 1 + \cos 2x$$

$$\rightarrow \frac{\sqrt{3} \sin x}{2} + \frac{\cos x}{2} + \frac{\cos x}{2} - \frac{\sqrt{3} \sin x}{2} =$$

$$= \cos^2 x + \sin^2 x + \cos^2 x - \sin^2 x \rightarrow \cos x = 2 \cos^2 x \rightarrow \cos x(2 \cos x - 1) = 0$$

$$\cos x = 0 \rightarrow \begin{cases} x_1 = 90^\circ + 360^\circ \cdot k \\ x_2 = 270^\circ + 360^\circ \cdot k \end{cases}$$

$$2 \cos x - 1 = 0 \rightarrow \cos x = \frac{1}{2} \rightarrow \begin{cases} x_1 = 60^\circ + 360^\circ \cdot k \\ x_2 = 300^\circ + 360^\circ \cdot k \end{cases}$$

10.-

$$a) \quad 4 \sin x - \sec x = 0 \rightarrow 4 \sin x \cdot \cos x - 1 = 0 \rightarrow 2 \sin 2x = 1 \rightarrow \sin 2x = \frac{1}{2}$$

$$\rightarrow \begin{cases} x_1 = 15^\circ + 180^\circ \cdot k \\ x_2 = 75^\circ + 180^\circ \cdot k \end{cases}$$

$$b) \quad \frac{\cos^2 x}{2 \cos x + \sin x} = \sin x \rightarrow \cos^2 x = 2 \cos x \cdot \sin x + \sin^2 x \rightarrow \cos 2x = \sin 2x$$

$$\rightarrow \begin{cases} x_1 = 22,5^\circ + 180^\circ \cdot k \\ x_2 = 112,5^\circ + 180^\circ \cdot k \end{cases}$$

$$c) \quad \frac{1}{\cos x + \sin x} + 2 \sin x = 2 \cos x \rightarrow \frac{2 \sin x \cdot \cos x + 2 \sin^2 x}{2 \cos^2 x + 2 \sin x \cdot \cos x} = 1$$

$$\rightarrow \frac{\sin x(\cos x + \sin x)}{\cos x(\cos x + \sin x)} = 1 \rightarrow \operatorname{tg} x = 1 \rightarrow \begin{cases} x_1 = 45^\circ + 360^\circ \cdot k \\ x_2 = 225^\circ + 360^\circ \cdot k \end{cases}$$

$$d) \quad \sin x(\sin x - 1) = 5 \cos^2 x - 4 \rightarrow \sin^2 x - \sin x = 5(1 - \sin^2 x) - 4$$

$$6 \sin^2 x - \sin x - 1 = 0$$

$$\rightarrow \sin x = -\frac{1}{3} \rightarrow \begin{cases} x_1 = 340^\circ 31' 44'' + 360^\circ \cdot k \\ x_2 = 199^\circ 28' 16'' + 360^\circ \cdot k \end{cases}$$

$$\rightarrow \sin x = \frac{1}{2} \rightarrow \begin{cases} x_1 = 30^\circ + 360^\circ \cdot k \\ x_2 = 150^\circ + 360^\circ \cdot k \end{cases}$$

$$e) \quad 2 \cos x - 1 = \sec x \rightarrow 2 \cos^2 x - \cos x - 1 = 0$$

$$\rightarrow \cos x = 1 \rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ \cdot k \\ x_2 = 180^\circ + 360^\circ \cdot k \end{cases}$$

$$\rightarrow \cos x = -\frac{1}{2} \rightarrow \begin{cases} x_1 = 120^\circ + 360^\circ \cdot k \\ x_2 = 240^\circ + 360^\circ \cdot k \end{cases}$$

$$f) \quad 2 \cos x + \sin x = 1 \rightarrow \sqrt{1 - \cos^2 x} = 1 - 2 \cos x \rightarrow 5 \cos^2 x - 4 \cos x = 0$$

$$\rightarrow \cos x(5 \cos x - 4) = 0 \rightarrow \cos x = \frac{4}{5} \rightarrow \begin{cases} x_1 = 36^\circ 52' 11,6'' + 360^\circ \cdot k \\ x_2 = 323^\circ 7' 48,4'' + 360^\circ \cdot k \end{cases}$$

$$g) \quad \sin x + \cos x = 0 \rightarrow \sin x = -\cos x \rightarrow \begin{cases} x_1 = 135^\circ + 360^\circ \cdot k \\ x_2 = 315^\circ + 360^\circ \cdot k \end{cases}$$