MATEMÁTICAS B 4º ESO. RECTA REAL, RADICALES Y LOGARITMOS

NOMBRE:____

1º) Calcular "x" en las siguientes expresiones:

$a)\ 2^x = 64 \qquad b)\ x = \log_8 5 \qquad c)\ 6^{2x} = 7 \qquad d)\ 3.\log 9 + 2.\log x^3 = 5$

$e)\ 4^x = 5 \qquad f)\ \log_x\left(\frac{1}{9}\right) = -2 \qquad g)\ \log_5 x = 3$

2º) Razonar si son ciertas las igualdades siguientes, sabiendo que $a \neq 1$:

a) $\frac{\log\left(\frac{1}{a}\right) - \log\sqrt{a}}{\log a^2} = \frac{3}{4}$ $\qquad b)\ \frac{\log\sqrt[3]{a} + \log\left(\frac{1}{a}\right)}{\log a^2}$

3º) Determina los intervalos siguientes y representalos:

$a)\ \{x \in R:\ 3 < x \leq 10\} \qquad b)\ x \geq 2 \qquad c)\ |x-5| \leq 3$

$d)\ |x+4| < 1 \qquad e)\ |x-3| > 2$

4º) Calcular:

$a)\ \sqrt[3]{2.\sqrt{5}\ .\ \sqrt[4]{2^2\ .5}} \qquad b)\ 2.\sqrt{18} - 3.\sqrt{20} - 5.\sqrt{125} + 7.\sqrt{32}$

5º) Racionalizar:

$a)\ \frac{\sqrt{2}+3}{\sqrt{5}} \qquad b)\ \frac{4+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \qquad c)\ \frac{3}{\sqrt[4]{5}}$

SOLUCIONES

1º

a) $2^x = 64 \Rightarrow 2^x = 2^6 \Rightarrow \boxed{x = 6}$

b) $x = \log_3 5 \Rightarrow \boxed{x = 1'4649}$ con calculadora.

c) $6^{2x} = 7 \Rightarrow \log 6^{2x} = \log 7 \Rightarrow 2x \cdot \log 6 = \log 7 \Rightarrow x = \dfrac{\log 7}{2 \cdot \log 6} = \underline{0'543}$

d) $3 \log 9 + 2 \cdot \log x^3 = 5 \Rightarrow \log 9^3 + \log (x^3)^2 = 5 \Rightarrow \log 729 + \log x^6 = 5 \Rightarrow$

$\Rightarrow \log (729 \cdot x^6) = 5 \Rightarrow 10^5 = 729 \cdot x^6 \Rightarrow x^6 = \dfrac{10^5}{729} \Rightarrow x^6 = 137'1742 \Rightarrow$

$\Rightarrow x = \sqrt[6]{137'1742} \Rightarrow \boxed{x = 2'27}$

e) $4^x = 5 \Rightarrow \log 4^x = \log 5 \Rightarrow x \cdot \log 4 = \log 5 \Rightarrow x = \dfrac{\log 5}{\log 4} \Rightarrow \boxed{x = 1'16}$

f) $\log_x \left(\frac{1}{9}\right) = 2 \Rightarrow x^{-2} = \frac{1}{9} \Rightarrow x^{-2} = \frac{1}{3^2} \Rightarrow x^{-2} = 3^{-2} \Rightarrow \boxed{x = 3}$

g) $\log_5 x = 3 \Rightarrow x = 5^3 \Rightarrow \boxed{x = 125}$

2º

a) $\dfrac{\log\left(\frac{1}{a}\right) - \log \sqrt{a}}{\log a^2} = \dfrac{\log 1 - \log a - \log a^{1/2}}{2 \cdot \log a} =$

$= \dfrac{-\log a - \frac{1}{2} \cdot \log a}{2 \cdot \log a} = \dfrac{-\frac{3}{2} \cdot \log a}{2 \cdot \log a} = \dfrac{-3/2}{2} = -\dfrac{3}{4}$ $\boxed{\text{FALSO}}$

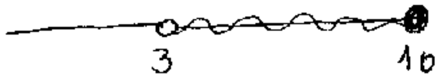
b) $\dfrac{\log \sqrt[3]{a} + \log\left(\frac{1}{a}\right)}{\log a^3} = \dfrac{\log a^{1/3} + \log 1 - \log a}{3 \cdot \log a} =$

$= \dfrac{\frac{1}{3} \cdot \log a - \log a}{3 \log a} = \dfrac{-\frac{2}{3} \cdot \log a}{3 \cdot \log a} = \dfrac{-2/3}{3} = \dfrac{-2}{9}$

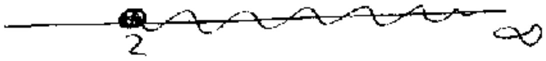
$\boxed{\text{VERDADERO}}$

3º

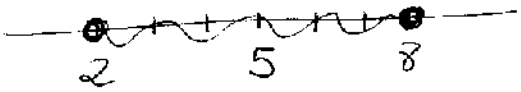
a) $\{x \in \mathbb{R} : 3 < x \leq 10\} \Rightarrow (3, 10]$



b) $x \geq 2 \Rightarrow [2, +\infty)$

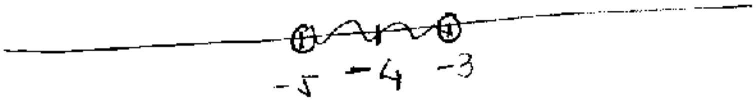


c) $|x-5| \leq 3$



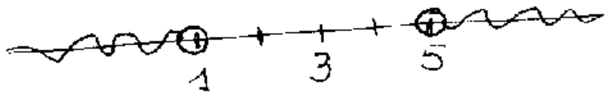
$[2, 8]$

d) $|x+4| < 1 \Rightarrow |x-(-4)| < 1$



$(-5, -3)$

e) $|x-3| > 2$



$(-\infty, 1) \cup (5, +\infty)$

4º

a) $\sqrt[3]{2 \cdot \sqrt{5}} \cdot \sqrt[4]{2^2 \cdot 5} = \sqrt[6]{2^2 \cdot 5} \cdot \sqrt[4]{2^2 5} =$

$= \sqrt[12]{(2^2 \cdot 5)^2} \cdot \sqrt[12]{(2^2 \cdot 5)^3} = \sqrt[12]{(2^2 \cdot 5)^5} = \sqrt[12]{2^{10} \cdot 5^5}$

b)

$2.\sqrt{18} - 3.\sqrt{20} - 5\cdot\sqrt{125} + 7\cdot\sqrt{32} =$

$= 2\cdot\sqrt{3^2 \cdot 2} - 3\cdot\sqrt{2^2 \cdot 5} - 5\cdot\sqrt{5^3} + 7\cdot\sqrt{2^5} =$

$= 2\cdot 3\cdot\sqrt{2} - 3\cdot 2\cdot\sqrt{5} - 5\cdot 5\cdot\sqrt{5} + 7\cdot 2^2\cdot\sqrt{2} =$

$= 6\sqrt{2} - 6\sqrt{5} - 25\sqrt{5} + 28\sqrt{2} = \boxed{34\sqrt{2} - 31\sqrt{5}}$

(5°) a) $\frac{\sqrt{2}+3}{\sqrt{5}} = \frac{(\sqrt{2}+3)\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \frac{\sqrt{10}+3\sqrt{5}}{\sqrt{5^2}} = \boxed{\frac{\sqrt{10}+3\sqrt{5}}{5}}$

b) $\frac{4+\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{(4+\sqrt{2})\cdot(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})\cdot(\sqrt{5}+\sqrt{2})} = \frac{4\sqrt{5}+4\sqrt{2}+\sqrt{10}+\sqrt{2}^2}{(\sqrt{5})^2-(\sqrt{2})^2} =$

$= \frac{4\sqrt{5}+4\sqrt{2}+\sqrt{10}+2}{5-2} = \boxed{\frac{4\sqrt{5}+4\sqrt{2}+\sqrt{10}+2}{3}}$

c) $\frac{3}{\sqrt[4]{5}} = \frac{3\cdot\sqrt[4]{5^3}}{\sqrt[4]{5}\cdot\sqrt[4]{5^3}} = \frac{3\cdot\sqrt[4]{5^3}}{\sqrt[4]{5^4}} = \boxed{\frac{3\cdot\sqrt[4]{5^3}}{5}}$