

Realiza la composición $(f \circ g)(x)$ y $(g \circ f)(x)$ de la siguiente pareja de funciones.

a) $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x-2}$

b) $f(x) = x^2 - x - 2$, $g(x) = \sqrt{2x-4}$

c) $f(x) = \frac{x+3}{x-3}$, $g(x) = \frac{x^2-1}{x}$

a) $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x-2}$

$$(f \circ g)(x) = f\left(\frac{1}{x-2}\right) = \frac{1}{\frac{1}{x-2}} = x-2 \quad , \quad (g \circ f)(x) = g\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}-2} = \frac{x}{1-2x}$$

b) $f(x) = x^2 - x - 2$, $g(x) = \sqrt{2x-4}$

$$(f \circ g)(x) = f(\sqrt{2x-4}) = (\sqrt{2x-4})^2 - \sqrt{2x-4} - 2 = 2x-4 - \sqrt{2x-4} - 2 = 2x - \sqrt{2x-4} - 6$$

$$(g \circ f)(x) = g(x^2 - x - 2) = \sqrt{2(x^2 - x - 2) - 4} = \sqrt{2x^2 - 2x - 8}$$

c) $f(x) = \frac{x+3}{x-3}$, $g(x) = \frac{x^2-1}{x}$

$$(f \circ g)(x) = f\left(\frac{x^2-1}{x}\right) = \frac{\frac{x^2-1}{x} + 3}{\frac{x^2-1}{x} - 3} = \frac{x^2+3x-1}{x^2-3x-1}$$

$$(g \circ f)(x) = g\left(\frac{x+3}{x-3}\right) = \frac{\left(\frac{x+3}{x-3}\right)^2 - 1}{\frac{x+3}{x-3}} = \frac{\frac{(x+3)^2 - (x-3)^2}{(x-3)^2}}{\frac{x+3}{x-3}} = \frac{12x}{(x-3)(x+3)} = \frac{12x}{x^2-9}$$